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Combining Immediate Fixed and Immediate Variable Annuities

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ABSTRACT

This paper explores the potential benefits of developing retirement income that considers both immediate fixed annuities (IFA) and immediate variable annuities (IVA) using a stochastic utility model combined with a scenario framework. Optimal annuity allocations vary considerably across household type, but certainty-equivalent retirement income increases by 20 percent, on average, when incorporating annuities. Total annuity allocations increase when both IFAs and IVAs are considered and retirees realize only approximately two-thirds of the benefits of annuitization when just one annuity type is considered. IVA allocations were typically higher than IFA allocations because most households already have a base level of fixed guaranteed income (through Social Security); therefore, IVAs can be a unique diversifier from a retirement-income perspective. Overall, this analysis strongly suggests retirees (and financial advisors) should consider annuities as part of a retirement-income strategy, and that they should consider different types of annuities to create the best possible plan.

INTRODUCTION

Diversification across different asset classes is an important part of building an efficient portfolio. Diversification across different asset classes and product types is also an important part of building an efficient income strategy for a retiree. Although many financial advisors use traditional investments (funds, exchange-traded funds, bonds, etc.) to build out retirement-income strategies, a significant body of research supports including guaranteed income products (annuities) as well. The cost of a forty-year retirement is much greater than the cost of a twenty-year retirement, and for many it is simply prudent to have some sort of income protection given the increased likelihood of living past the century mark. This paper explores the potential benefits of developing a retirement-income strategy using an IFA, which provides fixed payments for the life of the annuitant, or an IVA, where payments can vary depending on the performance of the underlying portfolio (often referred to as the subaccount).

The analysis uses a stochastic utility model combined with a scenario framework to isolate and determine household attributes best suited for annuitization. Households that should consider annuitization are generally those with conservative portfolios, lower levels of existing guaranteed income (i.e., Social Security benefits), higher initial withdrawal rates, higher subjective life expectancies, higher levels of shortfall risk aversion, and lower liquidity preferences. Building off the previous considerations, retiree households that are more likely to benefit from purchasing an IFA (versus an IVA) have relatively more-aggressive portfolios, lower levels of existing guaranteed income, lower equity return expectations, higher expected equity risk levels, and higher levels of shortfall risk aversion.

This analysis strongly suggests retirees (and financial advisors) should consider annuities as part of a retirement-income strategy, and that they should consider different types of annuities to create the best possible plan.

The percentage of households that should consider annuitizing increases when both IFAs and IVAs are considered during the product allocation process. Certainty-equivalent retirement income increases by 20 percent, on average, when incorporating annuities, although the gains differ significantly across households. Retirees realize only approximately two-thirds of the benefits of annuitization when just one annuity type is considered. IVAs tended to result in slightly higher allocations than IFAs because most households already have a base level of fixed guaranteed income (through Social Security); therefore, IVAs are a unique diversifier from a retirement-income perspective.

Overall, this analysis strongly suggests retirees (and financial advisors) should consider annuities as part of a retirement-income strategy and should consider multiple types of annuities to create the best possible plan.

LITERATURE REVIEW

Immediate annuities—both fixed and variable—are considered true annuities. A lump sum is irrevocably used to purchase an annuity contract that promises to pay out income for life. Approximately half of the purchasers will die prior to life expectancy, creating the income that will sustain the purchasers who will live longer. This is known as a mortality credit and can significantly improve the payout to those who are alive (versus self-annuitizing). Immediate annuities are available to retirees and generally are bought after the purchaser has finished accumulating retirement assets. Deferred annuities—both fixed and variable—are much more akin to traditional mutual funds, do not involve an irrevocable purchase, and can be appropriate investments for both retirees and those still accumulating retirement assets. Deferred annuities can be coupled with a guaranteed income rider and also can include a provision that serves as a potential substitute for an immediate annuity. Immediate annuities benefit from mortality credits but deferred annuities do not, which generally makes immediate annuities more efficient at generating income. This paper focuses on immediate annuities, both IFAs and IVAs.

Immediate annuities are one of the simplest and oldest strategies for creating a guaranteed lifetime income. In ancient Rome, contracts known as "annua" promised an individual a payment stream for a fixed term, or possibly for life, in return for an upfront payment (James 1947). Poterba (1997) notes that single-premium life annuities were available in the Middle Ages, and detailed records exist of special annuity pools known as tontines[1] that operated in Europe from the seventeenth century to the early twentieth century.

The majority of the research on annuities has focused on immediate nominal annuities, where the income stream for life remains constant for the entire payment duration (i.e., it does not increase with inflation). In one of the first papers that framed the potential benefits of annuitization, Yaari (1965) demonstrates how investors with no bequest motive should invest in annuities for retirement income using a model of intertemporal choice with lifetime uncertainty. Others have confirmed Yaari's conclusions. For example, Strawczynski (1999) also noted that individuals with no utility of bequest will annuitize all their liquid assets.

Even after relaxing the somewhat restrictive assumptions in Yaari's model, Davidoff et al. (2005) still note the potential benefit of full annuitization for a large set of individual preferences. The authors note that it may be optimal to annuitize up to two-thirds of a retiree's assets and fully annuitize if the net return on the annuity is greater than that of the reference asset. Additional research by Devolder and Hainaut (2006) and Kaplan (2006) explored the potential benefits of annuities in conjunction with different withdrawal strategies and portfolio allocations. Kaplan (2006) makes an important note that the long-term benefit of a nominal annuity will be materially affected by inflation levels following annuitization, because the value of a nominal annuity does not increase with inflation.

Although utility-based approaches are perhaps the most common way to measure the potential benefits of annuities, another approach based in part on the "safety-first rule" of Roy (1952) is popular, especially when determining the optimal income that can be withdrawn from a portfolio (discussed below). Under this approach, the objective is to maximize the probability of achieving a goal within a stochastic simulation (i.e., Monte Carlo) approach. For example, Milevsky et al. (2006) derive the optimal investment and annuitization strategies for a retiree using this approach and find (due largely to the binary nature of the approach) that investors should annuitize wealth only if they can cover fully the desired level of consumption.

A key pitfall when using the probability of success as an outcome metric in a financial plan is that it ignores the potential benefits (i.e., utility) a retiree would receive by achieving a goal. Because a success metric considers only two possible outcomes (success or failure), the degree to which success or failure occurs is ignored. Although Bayraktar and Young (2007) show that the results from maximizing expected utility of lifetime consumption under hyperbolic absolute risk aversion utility is identical to minimizing the probability of lifetime ruin because retirees can only succeed or fail, it is unlikely these results would hold for a wider array of scenarios.

There are obviously costs associated with an annuity. Longstaff (1995, 2001) argues that one should be compensated in equilibrium for illiquidity restrictions. In other words, all else being equal, a fixed income instrument that cannot be sold or subsequently repurchased over the life of the product should provide investors with a higher yield. Browne et al. (2003) develop a model for analyzing the liquidity premium required by the holder of an illiquid annuity and compute the yield (spread) needed to compensate for the utility welfare loss, which is induced by the inability to rebalance and maintain an optimal portfolio when holding an annuity; they suggest a premium of 45 to 145 basis points per annum as compensation for the inability to rebalance during a ten-year period.

The term "variable annuity" is relatively ambiguous, but for this analysis the term is used to describe an immediate annuity that provides income for life where the income varies depending on the performance of some subaccount and where the annuitization decision itself is irrevocable. Other types of deferred variable annuities are revocable and offer guaranteed lifetime income, such as annuities that contain a guaranteed lifetime withdrawal benefit (GLWB). Revocability is obviously an attractive feature with an annuity; however, it typically results in lower expected lifetime income. For example, Goodman and Richardson (2016) compare IVAs and deferred annuities with GLWB riders and find that a strategy that includes IVAs would have outperformed a deferred annuity with a GLWB strategy for a retiree who desired to have lifetime income with inflation protection, maintain some liquidity, and have the potential for an estate.

Although most research focuses on fixed annuities, there is a growing body of research on variable annuities. For example, Brown et al. (2001) find that a variable payout annuity is more attractive than a fixed annuity with benefits linked to inflation for consumers with moderate risk aversion. Ibbotson et al. (2007) specifically note that combining immediate fixed and variable life annuities with conventional investment instruments, such as mutual funds, is the optimal retirement-income solution. However, the authors do not provide guidance on which households should consider annuities, and if so which type. Research by Horneff et al. (2010) explored the potential benefits of variable annuitization using a dynamic utility model and find that consumer well-being could be enhanced substantially if equities were integrated more fully into retirees' annuity and liquid wealth portfolios.

Despite the fact annuities are well-known, and that research has shown them to be a valuable benefit for retirees, they are not widely used by retirees. This lack of utilization has been noted as a "puzzle" by some. For example, Modigliani (1986) noted the existence of the "annuity puzzle" in his Nobel acceptance speech when he stated: "It is a well-known fact that annuity contracts, other than in the form of group insurance through pension systems, are extremely rare. Why this should be so is a subject of considerable current interest. It is still ill-understood." Brown (2001), Milevsky and Young (2007), and Purcal and Piggott (2008) each noted the relatively low voluntary annuitization rate; and Bhojwani (2011) noted that in a survey that 54 percent of Americans ages forty-four to seventy-five expressed distaste for the word "annuity."

Kahneman and Tversky (1979) demonstrated that people do not always behave according to the expected utility paradigm, which may explain some of the lack of annuity demand; however, other factors likely are involved in a retiree's decision whether or not to annuitize a portion of wealth. A variety of different frameworks have been introduced to help explain the annuity puzzle, such as the lack of fairly priced annuities, inflation risk, bequest motives, the impact of Social Security and pension benefits, the presence of uncertain medical expenditures, rare events, and flexibility. For example, to counter Yaari (1965), Bernheim (1991) notes that a bequest motive can entirely mitigate the demand for annuities.

The annuity puzzle is not just a domestic issue. Johnson et al. (2004) report that, in the United States, private annuities finance less than 1 percent of household income for people older than age sixty-five. They also observe that private annuities are purchased by only 5 percent of people older than sixty-five. James and Song (2001) find similar results for other countries, such as Canada, the United Kingdom, Switzerland, Australia, Israel, Chile, and Singapore. Using U.K. microeconomic data to determine the empirical determinants of voluntary annuity market demand, Inkmann et al. (2011) find that annuity market participation increases with financial wealth, life expectancy, and education and decreases with other pension income and a possible bequest motive for surviving spouses.

This paper adds to the existing literature of annuitization by focusing on the potential benefits of jointly considering IFAs and IVAs across a wide variety of assumed household types using a stochastic utility model with a particular focus on which types of households would derive the most utility from an IVA or IFA.

ANALYSIS

The optimal retirement-income strategy (e.g., annuity allocation) is determined using an approach based on the constant relative risk aversion (CRRA) utility function, shown in equation (1), where the amount of utility U received varies depending on level of consumption c and level of investor risk aversion γ:

$$U(c)=\frac{c^{1-\gamma}}{1-\gamma} \quad (1)$$

Implied within the CRRA utility function is the law of diminishing marginal utility, whereby negative outcomes (especially extreme negative outcomes) are weighted more heavily than positive outcomes. The penalty associated with bad retirement outcomes can vary based on a variety of factors.

The specific utility approach used in this paper is a modified version of the approach introduced by Blanchett and Kaplan (2013). The two primary adjustments for this model are the introduction of a liquidity preference factor as well as a bequest preference factor (i.e., it considers both income and residual wealth when determining the optimal strategy). The model is described in detail in appendix 1.

The initial annual portfolio need is assumed to be \$50 (i.e., \$50,000, because most amounts are assumed to be thousands of dollars) for all simulations, although subsequent withdrawals vary depending on additional scenario assumptions. The initial balance varies across runs, which results in different initial portfolio withdrawal rates (i.e., funded status levels). Withdrawals always are assumed to take place at the beginning of the year. Taxes are ignored for the analysis.

The initial portfolio balance is determined by dividing the initial annual portfolio need (which is always \$50) by the initial portfolio withdrawal rate. Initial portfolio withdrawal rates are used as the metric of retirement readiness (or fundedness) given their predominance in the retirement-income literature. For example, if the initial withdrawal rate is assumed to be 4 percent, the initial portfolio balance would be \$1,000 (i.e., \$1 million, \$50 ÷ 4% = \$1,250). Social Security retirement benefits also are based on the initial annual portfolio need. For example, if Social Security is assumed to be 50 percent of the total retirement income, the initial annual Social Security retirement benefit would be $50, for a total retirement-income need of $100. Social Security retirement benefits are assumed to increase annually by inflation, although the rate by which the total retirement benefit increases varies across scenarios.

Three types of asset returns are generated for the analysis: inflation, bonds, and stocks. All returns are assumed to be normally distributed and the mean and standard deviation varies by scenario. Although returns are not perfectly normally distributed, they are approximately so, especially at an annual frequency. The correlations among these asset classes are assumed to be zero, which is consistent with actual historical correlations (see appendix 2). The return assumptions are unconditional in nature and do not necessarily reflect current market conditions.

Mortality rates for the analysis are based on the Society of Actuaries Individual Annuity Mortality Table (2012 IAM) with improvement to year 2019. The actual rates from the mortality table are used for annuity pricing, but a Gompertz approach is used when estimating survival probabilities for the retiree household in order to incorporate subjective life expectancy assumptions. The specific Gompertz approach and key parameters are explained more fully in appendix 3.

The retiree household for the analysis is assumed to be at retirement and planning to purchase an annuity straight away with payments that commence immediately. Annuity payout rates are determined using a pricing model based on the mortality-weighted net present value of the expected benefits. The research for this paper used a pricing model, rather than actual annuity quotes, to ensure consistency across assumptions; however, the model is compared to actual quotes for quality control purposes.

All annuities are assumed to be life-only (i.e., without any type of period certain or cash refund provision). The discount rate for the IFA pricing calculation is constant and assumed to be 5 percent. This is higher than the assumed average 4-percent bond return for the portfolio to reflect differences in yield associated with the assumed bond-pricing index, which is long-duration AAA bonds, versus the assumed portfolio fixed income asset class, which is intermediate government bonds. The base assumed yield difference between these two bond asset classes (i.e., 1 percent) is lower than historical averages. For example, the total return (yield) on the Bloomberg Barcap US Corporate AAA Long Index has been 1.3 percent (1.69 percent) higher than the Bloomberg Barcap US Government Intermediate Index from 1974 to 2018; therefore, a 1-percent difference is a relatively conservative (annuity pricing) assumption.

The discount rate for the IVA pricing calculation is also constant for the pricing calculation and based on the assumed interest rate (AIR). If realized returns exceed the AIR, the IVA payments will increase, and if returns underperform the AIR, payments will decrease. Low AIRs (e.g., 3 percent) result in lower initial income but an opportunity for significant increases, and higher AIRs (e.g., 5 percent) result in higher initial income but a higher probability of a future decrease. An AIR of 4 percent was assumed for the analysis, along with an equity allocation of 10 percent for the IVA for the base analysis; both these assumptions are evaluated below for robustness. Final estimates of payouts using an annuity pricing model, estimated by calculating the mortality-weighted net present value, are reduced by 5 percent to reflect a 5-percent load.

The annuity pricing model does a relatively good job tracking IFA payouts if an assumed 5-percent discount rate is substituted for current bond yields. For example, as of May 2019, the yield on the Bloomberg Barcap US Corporate AAA Long Index was 3.61 percent. Using a 3.61-percent discount rate in the IFA annuity pricing model results in an estimated payout of 5.29 percent for a sixty-five-year-old couple with life-only payments and a 100-percent continuation benefit. This is similar to the average payout rate of 5.27 percent based on the nineteen quotes obtained from CANNEX on May 27, 2019 (using the same pricing assumptions). Note, the pricing model for the analysis intended to target the average annuity payout, not the best possible payout available. Annuities with higher payouts obviously will be more attractive (and result in higher annuity allocations) and therefore payout rates are varied as part of the analysis.

Although realized bond returns are varied for the analysis, the IFA discount rate is held constant at 5 percent. This is because the future returns on a portfolio are unknown at the time of annuity purchase, but the payout rate for the annuity is certain. If future portfolio returns are higher than the bond-pricing component (i.e., yields rise), purchasing the annuity is likely to be less advantageous; conversely, if yields fall, purchasing the annuity is likely to be more advantageous. However, this requires an ability to correctly predict market changes and has other implications for retirees (e.g., optimal equity allocation, optimal initial withdrawal rate, etc.).

To avoid using a single set of parameters or household attributes to determine the optimal annuity allocations (even though this is relatively common in annuity research), virtually all the key assumptions and parameters in the model were varied. Using a single set of assumptions (i.e., targeting the average retiree household) could result in conclusions that are not robust across the varied circumstances and preferences that exist among retiree households or the variety of assumptions that advisors use in financial plans. This approach is designed to tease out broad differences that may change demand for the respective products (IFAs and IVAs) rather than provide guidance for a single household. Each variable has low, moderate, and high values.

VARIABLE ASSUMPTIONS

Twenty-two variable assumptions are considered in the analysis, including assumptions about client scenarios, portfolios, annuities, and client preferences. Each of these twenty-two variables is described below.

Client Scenario Assumptions

Retirement age: The retiree household is assumed to be a couple with spouses of the same age. The low, mid, and high retirement ages tested are sixty, sixty-five, and seventy, respectively.

Portfolio equity allocation: The equity allocation of the portfolio is assumed to be both exogenous from the annuity decision and to remain constant for the duration of retirement (i.e., it does not follow any type of glide path). In reality, the annuity purchase decision likely should impact the portfolio risk level and is likely to vary throughout retirement. For example, an IFA is generally considered a relatively bond-like asset, so an individual who purchases an IFA might increase the risk of the portfolio in an attempt to maintain the aggregate risk of the investor's total wealth. This is obviously an important consideration for financial advisors, but it is not clear to what extent this occurs today and attempting to incorporate this effect would complicate the analysis. The effective risk of IFAs and IVAs, however, is explored below to provide guidance on how these annuities should be incorporated. The low, mid, and high equity allocations tested are 5 percent, 40 percent, and 75 percent, respectively.

Social Security retirement benefits (as a percentage of total retirement income): The amount of existing guaranteed income has a significant impact on the potential of annuitization. For the analysis, the household is assumed to have real pension benefits (i.e., benefits linked to inflation). For simplicity purposes these benefits are referred to as "Social Security retirement benefits," although the pension benefits included in this model are slightly different than actual Social Security retirement benefits (e.g., the benefit is constant for the entire simulation and is not based on the survival composition of the retiree household). Social Security benefits are assumed to be some percentage of the initial total annual retirement-income need. Although these benefits grow with inflation, the total annual retirement need may grow at a different rate (e.g., inflation plus or minus 1 percent) that would cause the benefits to comprise a growing or shrinking share of the monies used to fund the retirement-income goal. Social Security retirement benefits are assumed to cover 5 percent, 40 percent, or 75 percent of the total retirement-income need for the low, mid, and high values, respectively.

Portfolio initial withdrawal factor: The initial withdrawal rate varies depending on the age at retirement, because older retirees generally can have higher initial withdrawal rates due the shorter expected duration of retirement. The initial withdrawal rate is determined by dividing the retiree age by an initial withdrawal factor. For example, if the retiree couple was sixty-five years old and the portfolio initial withdrawal factor was 15, the initial withdrawal rate would be 4.33 percent (65 ÷ 15 = 4.33). Initial portfolio withdrawal factors are assumed to be 20, 15, or 10 for the low, mid, and high values, respectively.

Percentage of retirement need that is flexible: Although retirement research has commonly treated the entire retirement need as fixed, retirees typically have some level of flexibility regarding spending needs. Therefore, for this analysis, the retirement need is decomposed into two parts: nondiscretionary and discretionary. The nondiscretionary portion is assumed to increase each year by inflation regardless of the retiree's funded status or portfolio performance. The discretionary portion is assumed to evolve each year with the performance of the portfolio, where the portfolio withdrawal is based on some percentage of the portfolio balance (similar, in spirit, to the required minimum withdrawal calculation). The approach to determining the annual discretionary withdrawal rate is outlined in appendix 4. The percentage of the total retirement need that is assumed to be discretionary is 0 percent, 25 percent, and 50 percent for the low, mid, and high values, respectively.

Real change in retirement-income need: Although retirement-income need commonly is assumed to increase with inflation (i.e., is constant in real terms), evidence suggests that retiree spending tends to decline over time (e.g., Blanchett 2014). For the analysis, the real retirement need is assumed to change by −1 percent, 0 percent, and +1 percent per year, for the low, mid, and high values, respectively. A real change of 0 percent would imply the retirement-income need increases each year in retirement with inflation, which is the most common assumption in retirement research.

Change in subjective life expectancies: As noted above, mortality rates for the analysis are based on the Society of Actuaries Individual Annuity Mortality Table (2012 IAM) with improvement to year 2019. A Gompertz approach was used to determine mortality rates for the retiree couples, as detailed in appendix 3, in order to incorporate subjective mortality estimates into the model. The modal value is adjusted in the Gompertz equation by −4, 0, and +4 to reflect low, mid, and high subjective life expectancies, respectively. The same adjustment to the model value is assumed for both members of the household; therefore, this adjustment should be viewed as an aggregate subjective life expectancy adjustment rather than an individual adjustment. The annuity pricing model is not affected by subjective life expectancy, although the potential benefits of annuitization obviously will be affected.

Portfolio Assumptions

Portfolio fee: Annual portfolio fees of 0.1 percent, 0.5 percent, or 1.0 percent are assumed for the low, mid, and high values, respectively, and are assessed against whatever monies exist in the portfolio that are not annuitized.

Annual equity arithmetic return: Equity returns of 5 percent, 8 percent, and 11 percent are assumed for the low, mid, and high values, respectively.

Annual equity standard deviation: The annual standard deviation for equity returns for the low, mid, and high values are 16 percent, 20 percent, and 24 percent, respectively.

Annual bond return: Bond returns of 3 percent, 4 percent, and 5 percent for the low, mid, and high values, respectively, are considered. The assumed bond asset class is intermediate government bonds.

Annual bond standard deviation: The annual standard deviation for bond returns for the low, mid, and high values are 4 percent, 6 percent, and 8 percent, respectively.

Annual inflation: Annual inflation levels of 1.5 percent, 2.5 percent, and 3.5 percent for the low, mid, and high values, respectively, are considered.

Annual inflation standard deviation: The annual standard deviation for inflation for the low, mid, and high values are 2 percent, 3 percent, and 4 percent, respectively.

Annuity Assumptions

Change in immediate fixed annuity payout: The assumed payout for the IFA is changed by -15 percent, 0 percent, and +15 percent for the low, mid, and high values, respectively.

Change in immediate variable annuity payout: The assumed payout for the IVA is changed by -15 percent, 0 percent, and +15 percent for the low, mid, and high values, respectively.

Immediate variable annuity expense ratio: The expense ratio for the IVA is assumed to be 0.1 percent, 0.5 percent, or 1.0 percent, for the low, mid, and high values, respectively.

Assumptions About Client Preferences

The following assumptions relate to the utility model introduced in appendix 1.

Shortfall risk aversion: Low, mid, and high values are assumed to be 1.01, 2, and 4, respectively.

Bequest preference: Low, mid, and high values are assumed to be 0, 0.5, and 1.0, respectively.

Liquidity preference: Low, mid, and high values are assumed to be 0 percent, 10 percent, and 20 percent, respectively.

Risk aversion coefficient: Low, mid, and high values are assumed to be 0.99, 0.5, and 0.25, respectively.

Real subjective discount rate: Low, mid, and high values are assumed to be 0 percent, 2 percent, and 4 percent, respectively.

There are more than 31 billion different potential combinations across these 22 different assumptions. Instead of modeling all 31 billion combinations, 10,000 different scenarios are generated where a value (among the three) is randomly selected for each variable.[2]

The annuity allocation is assumed to never exceed 50 percent of the portfolio. Although many retiree households may benefit from higher levels of annuitization, it is unrealistic to expect that many households would be willing to annuitize a major portion of retirement savings.

The optimal annuity allocation is determined in 10-percent increments up to the 50-percent total maximum possible allocation. This results in six allocations for a single product (0 percent, 10 percent, 20 percent, 30 percent, 40 percent, and 50 percent) and a total of twenty-one different potential combinations when both products are considered jointly, as shown in table 1. Table 1 shows the optimal annuity determined by the utility model fully outlined in appendix 1 for the scenario that uses all "mid" assumptions across the twenty-two variables.

Table 1

MODEL EXAMPLE

		IVA Allocation					
		0%	10%	20%	30%	40%	50%
IFA Allocation	0%	49.36	50.39	51.12	51.69	52.15	52.53
	10%	50.18	51.21	51.96	52.56	53.05	n/a
	20%	50.82	51.88	52.66	53.29	n/a	n/a
	30%	51.31	52.42	53.25	n/a	n/a	n/a
	40%	51.68	52.86	n/a	n/a	n/a	n/a
	50%	51.95	n/a	n/a	n/a	n/a	n/a

Best IFA Only Allocation (51.95) *Best IVA Only Allocation* (52.53) *Best Combined Allocation* (53.29)

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In this case, the optimal product allocation resulting in the highest level of utility occurs when the IVA allocation is 30 percent and IFA allocation is 20 percent (utility = 53.29). The utility is lowest for the scenario with no assumed annuitization (49.36). Considering only the IFA or only the IVA was better than not considering any annuity at all in this particular case; the IVA-only scenario generated slightly more utility than the IFA-only scenario (52.53 versus 51.95, respectively). This suggests if the retiree household is willing to consider purchasing only one of the two products, the IVA would be the better option.

A retiree can generate additional guaranteed income in other ways, such as delaying Social Security retirement benefits, accessing another private or public defined benefit plan, and purchasing a different type of annuity. A key assumption in this analysis is that the client is interested only in purchasing an IFA and an IVA with income that would commence immediately. This is obviously a restrictive set of assumptions, but attempting to incorporate other aspects of the annuity decision (e.g., considering a deferred income annuity, coordinating the purchase with delaying Social Security retirement benefits, etc.), materially complicates the analysis. The intentionally narrow scope of the analysis enables an in-depth look at the potential benefits associated with diversified immediate annuitization.

RESULTS

This section details the results for the 10,000 different scenarios. Recall that for the analysis, one of three values (low, mid, and high) was randomly selected for each of the twenty-two potential variables. Annuity allocations are expected to vary considerably across scenario assumptions; however, this approach allows us to determine which factors should affect the demand for IFAs versus IVAs.

Table 2 includes the distribution of product allocations across the 10,000 considered scenarios, including the allocations when both products are considered as well as if only one product was available.

Focusing on the scenarios where both products are available, the average total annuity allocation was 41.96 percent. This is a rather significant allocation given that the maximum possible allocation is 50 percent, and it suggests that many retirees would be better having more of their wealth annuitized, which is consistent with most research on this topic. The total annuity allocation is larger when both products are available. For example, the average single product allocations are 33.01 percent and 34.43 percent when considering the IFA-only and IVA-only, respectively, versus 41.96 percent when both are available.

This is a rather significant allocation given that the maximum possible allocation is 50 percent, and it suggests that many retirees would be better having more of their wealth annuitized, which is consistent with most research on this topic.

The allocations were typically "all-or-none," even when both products were available. Only 26.5 percent of scenarios had allocations to both products, which is 30.6 percent of all scenarios including an annuity allocation, which is less than the total allocations to either product individually.

Table 3 provides context as to how the annuity allocations varied for each of the 22 variables for the low, mid, and high values. Panel A shows the total annuity allocation to both products when both are available. Panel B shows the percentage of the total annuity allocation the IFA represents when both are available. Panel C shows the total annuity allocation when only IFAs are available. Panel D shows the total annuity allocation when only IVAs are available.

Table 2

DISTRIBUTION OF PRODUCT ALLOCATIONS

BOTH AVAILABLE

		IVA Allocation					
		0%	10%	20%	30%	40%	50%
IFA Allocation	0%	13.2	0.8	0.6	0.6	0.5	30.2
	10%	0.6	0.2	0.1	0.1	3.4	n/a
	20%	0.6	0.1	0.1	4.4	n/a	n/a
	30%	0.6	0.1	5.4	n/a	n/a	n/a
	40%	0.7	7.5	n/a	n/a	n/a	n/a
	50%	30.0	n/a	n/a	n/a	n/a	n/a

	IFA Only			IVA Only	
Allocation	0%	30.3	Allocation	0%	25.6
	10%	1.7		10%	2.9
	20%	1.8		20%	2.7
	30%	2.0		30%	2.7
	40%	2.2		40%	2.5
	50%	62.0		50%	63.6

Table 3

AVERAGE ANNUITY ALLOCATIONS (%) BY VARIABLE

	Panel A			Panel B			Panel C			Panel D		
	Total Annuity Allocation			IFA, % of Total, Both Available			IFA Allocation, Only IFA Available			IVA Allocation, Only IVA Available		
Variable	**Low**	**Mid**	**High**	**Low**	**Mid**	**High**	**Low**	**Mid**	**High**	**Low**	**Mid**	**High**
Retirement Age	43	42	41	50	52	52	34	33	32	35	34	34
Equity Allocation	45	42	39	41	52	62	38	32	30	39	34	30
Social Security (% of Total Income)	47	42	37	68	49	33	45	33	21	42	31	30
Portfolio Initial Withdrawal Rate Factor	39	42	45	53	54	48	30	33	36	29	34	40
% of Need that is Flexible	41	43	41	56	49	49	34	34	31	33	36	33
ΔRetirement Income Real Need	40	42	44	51	52	51	31	33	36	32	34	37
ΔSubjective Life Expectancies	37	43	46	53	51	50	28	34	37	29	35	39
Portfolio Fee	40	42	44	50	52	51	30	33	36	32	34	38
Equity Return	41	41	43	89	48	18	40	33	26	25	36	42
Equity Standard Deviation	41	42	42	42	52	61	31	34	35	36	35	32
Annual Bond Return	44	42	39	51	52	52	37	33	29	38	34	31
Bond Standard Deviation	41	42	43	50	52	52	32	33	34	34	34	35
Annual Inflation	42	42	42	52	51	52	33	33	33	34	34	35
Inflation Std Dev	42	42	42	51	51	52	33	33	33	35	34	34
ΔIFA Payout Rate	38	42	46	37	51	64	24	33	42	34	34	35
ΔIVA Payout Rate	39	42	45	67	52	38	33	33	33	27	35	42
IVA Expense Ratio	43	42	40	46	52	56	33	33	33	37	34	32
Shortfall Risk Aversion	40	42	45	47	51	55	29	32	37	30	34	38
Bequest Preference	43	43	40	57	49	48	36	33	30	35	35	33
Liquidity Preference	46	42	37	52	51	51	38	33	28	39	35	30
Risk Averson Coefficient	42	41	43	44	50	61	31	32	36	36	35	32
Real Subjective Discount Rate	43	42	41	52	51	51	35	33	32	35	33	32
Low Target	40	40	40	45	45	45	30	30	30	30	30	30
High Target	45	45	45	55	55	55	38	38	38	38	38	38
Min	40%	40%	40%	45%	45%	45%	30%	30%	30%	30%	30%	30%
Max	45%	45%	45%	55%	55%	55%	35%	35%	35%	35%	35%	35%

Panel A, which is the total annuity allocation, provides some perspective on who should consider purchasing an annuity (i.e., which types of households would gain the most utility from additional guaranteed income). Annuity allocations are higher for households with more-conservative portfolios, lower levels of existing guaranteed income (i.e., Social Security retirement benefits), higher initial withdrawal rates, higher subjective life expectancies, higher levels of shortfall risk aversion, and lower liquidity preferences. These are the households that are likely to benefit more from higher annuity allocations.

Panel B, which includes the IFA allocation as a percentage of total annuity allocation when both types of annuities are available, provides insight into how the scenario assumptions impact the decision to purchase an IFA or IVA. Beyond the obvious impacts (e.g., lower IFA payouts reduce demand for the IFA relative to the IVA), households that likely benefit more from purchasing an IFA versus an IVA have more aggressive portfolios, lower levels of existing guaranteed income, lower expected equity returns, higher levels of expected equity risk (standard deviation), higher levels of shortfall risk aversion, lower bequest preferences, and higher risk aversion coefficients.

The individual product allocations in panels C and D are relatively consistent with the results in panels A and B. Certain household attributes make either type of annuity attractive (e.g., lower levels of existing guaranteed income), and certain other assumptions create relative demand (e.g., the IVA becomes

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Table 4

ORDINARY LEAST SQUARES RESULTS ON PRODUCT ALLOCATIONS

	Panel A		Panel B		Panel C		Panel D		Panel E	
	Total Allocation Both Available		IFA–IVA Both Available		Total Allocation IFA Only		Total Allocation IVA Only		IFA–IVA Only One	
	Coeff	t stat	Coeff	t stat	Coeff	t stat	Coeff	t stat	Coeff	t stat
Intercept	0.602***	20.631	0.521***	11.773	0.737***	23.322	0.380***	11.802	0.357***	8.789
Retirement Age	−0.002***	−5.469	0.001*	2.151	−0.001**	−2.787	−0.001*	−2.551	0.000	-0.146
Equity Allocation	−0.084***	−16.258	0.246***	31.350	−0.111***	−19.869	−0.131***	-23.051	0.020**	2.804
Social Security (% of Total Income)	−0.148***	−28.574	−0.433***	−55.201	−0.335***	−59.759	−0.170***	-29.736	-0.165***	-22.920
Portfolio Initial Withdrawal Rate Factor	0.993***	13.924	−1.573***	−14.542	0.853***	11.046	2.079***	26.426	-1.226***	-12.336
% of Need that is Flexible	0.008	1.102	−0.110***	−9.880	−0.046***	−5.866	0.001	0.113	-0.047***	-4.651
ΔRetirement Income Real Need	2.058***	11.385	−0.606*	−2.211	2.412***	12.318	2.961***	14.849	-0.549*	-2.180
ΔSubjective Life Expectancies	0.010***	21.799	−0.004***	−6.098	0.010***	20.148	0.012***	24.515	-0.002***	-3.746
Portfolio Fee	4.846***	12.031	−1.009	−1.653	6.059***	13.889	6.064***	13.650	-0.005	-0.009
Equity Return	0.316***	5.217	−10.290***	−112.170	−2.351***	−35.887	2.800***	41.968	-5.152***	-61.136
Equity Standard Deviation	0.134**	2.940	2.081***	30.125	0.461***	9.333	−0.561***	-11.169	1.022***	16.101
Annual Bond Return	−2.647***	−14.584	0.373	1.354	−3.780***	−19.231	−3.361***	-16.790	-0.419	-1.658
Bond Standard Deviation	0.156	1.718	−0.063	−0.456	0.196*	1.987	0.166	1.657	0.029	0.233
Annual Inflation	0.173	0.951	−0.832**	−3.025	0.090	0.458	0.577**	2.882	-0.487	-1.925
Inflation Standard Deviation	−0.189	−1.037	0.172	0.625	−0.304	−1.544	−0.102	-0.507	-0.202	-0.799
ΔIFA Payout Rate	0.222***	18.328	0.813***	44.223	0.563***	42.917	0.005	0.362	0.558***	33.083
ΔIVA Payout Rate	0.229***	18.819	−0.776***	−42.063	−0.018	−1.337	0.509***	37.914	-0.527***	-31.061
IVA Expense Ratio	−2.848***	−7.041	8.650***	14.103	0.213	0.487	−5.864***	-13.144	6.077***	10.786
Shortfall Risk Aversion	0.024***	13.392	0.031***	11.295	0.039***	19.899	0.040***	19.771	0.000	-0.183
Bequest Preference	−0.012***	−6.799	−0.041***	−14.983	−0.031***	−15.760	−0.011***	-5.384	-0.020***	-7.991
Liquidity Preference	−0.043***	−23.854	−0.005	−1.636	−0.051***	−25.964	−0.043***	-21.579	-0.008**	-3.101
Risk Averson Coefficient	0.004*	2.422	0.080***	29.037	0.025***	12.798	−0.018***	-9.007	0.043***	17.082
Real Subjective Discount Rate	−0.014***	−7.638	0.005	1.809	−0.015***	−7.331	−0.016***	-7.868	0.001	0.530
Observations	10,000		10,000		10,000		10,000		5,000	
R^2	29.01%		69.44%		50.77%		44.69%		41.65%	
Adjusted R^2	28.85%		69.38%		50.66%		44.56%		41.52%	

**** significant at 0.1% level, ** significant at 1% level, * significant at 5% level*

more attractive when equity returns are higher, which makes the IFA less attractive).

Next, to better isolate the drivers of the respective product allocations, a series of ordinary least squares (OLS) regressions were performed. The independent variables in the regressions are the test variables considered for the scenario. Five dependent variables are considered, as shown in table 4: the total product allocation when both annuities are available (panel A), the IFA allocation minus the IVA allocation when both products are available (panel B), the total allocation to the IFA when only the IFA is available (panel C), the total allocation to the IVA when only the IVA is available (panel D), and the allocation to the IFA minus the allocation to the IVA when only one of each product is available for a given scenario (panel E). The results of the regressions are included in table 4.

The regression results in table 4 are consistent with the average product allocations shown in table 3, which is not surprising. Table 4, though, provides a way to more easily quantify the relative effects on the annuity allocations. A few variables are not statistically significant when looking at the combined product decision, such as inflation, but they are important when selecting between types of annuities (e.g., higher inflation makes IVAs relatively more attractive given the positive assumed equity risk premium).

The most interesting, and perhaps most useful, results in table 4 are when the annuity allocations are contrasted (e.g., in panels B and E). The signs of the coefficients are consistent for all variables in panels B and E; however, there are notable differences in coefficient sizes. For example, the product allocation generally increases for retirees with lower levels of existing guaranteed income, yet IFAs are considerably more attractive for those households without a base of guaranteed income (compared to IVAs). This suggests households generally should seek out some minimum base level of fixed guaranteed income, which may be covered from Social Security retirement benefits, and only consider IVAs after some minimum target level of income is achieved.

Next, utility gains associated with considering annuities were explored, as shown in table 5, assuming a maximum potential gain in utility (i.e., an increase in certainty-equivalent income) of 25 percent. The minimum potential benefit of the product allocation is 0 percent (because these households would be assumed to not purchase the annuity).

The results in table 5 show that there are significant potential benefits associated with considering both IFAs and IVAs for retirees, whereby certainty-equivalent retirement income is increased by approximately 20 percent for the monies that could be used to annuitize. A retiree household captures only about two-thirds of the maximum possible utility benefits associated with annuitization if only one product is considered, the IFA or the IVA.

The results in table 5 also suggest that if the retiree household is willing to select only a single product (i.e., they are mutually exclusive), the IVA would be slightly more advantageous. This benefit can be attributed to the fact that these households already are assumed to have some existing fixed guaranteed income. If Social Security benefits are removed, the IFA would be more attractive; therefore, the level of existing fixed guaranteed income has a substantial effect on the optimal immediate annuity type.

ADDITIONAL IMMEDIATE VARIABLE ANNUITY CONSIDERATIONS

The primary analysis assumed the equity allocation of the IVA was 100 percent and that the AIR was 4 percent. Here, these assumptions are explored at greater depth to understand how varying them could affect results. The risk implications associated with purchasing the product (i.e., the effective risk from a total wealth perspective) is also explored.

First, the impact of different IVA equity allocations and AIRs is explored. The same base 10,000 simulations are used, but five different AIR values (3.0 percent, 3.5 percent, 4.0 percent, 4.5 percent, and 5.0 percent) and five different IVA equity allocations (0 percent, 25 percent, 50 percent, 75 percent, and 100 percent) are assigned randomly to the scenarios. Again, the optimal allocation is determined.

For this analysis, the focus is on how the annuity allocation changes for these scenarios compared to the base scenario where the equity allocation is 100 percent and the AIR is 4 percent. This provides some perspective as to how optimal the base scenario assumptions are. The comparison results are shown in table 6, where panel A is a comparison of the allocations to the IVA when both products were available, and panel B is a comparison when only the IVA was available.

Table 5
UTILITY IMPACT

		% Increase in Certainty-Equivalent Income (vs. No Annuitization)					% of Maximum Possible Utility Captured from Considering only One Annuity Type	
		IFA Only	IVA Only	Both			IFA Only	IVA Only
Percentile	5th	0.00	0.00	0.00	Percentile	5th	0.00	0.00
	25th	0.00	0.00	8.00		25th	0.00	0.00
	Median	11.40	13.79	20.39		Median	73.18	65.73
	Average	12.24	13.25	16.31		Average	55.78	56.69
	75th	25.00	25.00	25.00		75th	100.00	100.00
	95th	25.00	25.00	25.00		95th	100.00	100.00

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Table 6

IVA PRODUCT ASSUMPTIONS, DIFFERENCE VERSUS BASE CASE (4% AIR, 100% EQUITY)

		IVA Equity Allocation (%)					
Panel A: IVA Allocation Difference, Both Available							
		0	**25**	**50**	**75**	**100**	**Average**
AIR (%)	**3.0**	−13.14	−10.88	−3.83	−0.97	0.78	**−5.61**
	3.5	−17.78	−10.05	−4.56	−1.61	0.42	**−6.72**
	4.0	−16.94	−10.75	−6.28	−1.46	0.00	**−7.09**
	4.5	−16.69	−10.37	−7.04	−2.13	−0.33	**−7.31**
	5.0	−16.22	−10.47	−5.61	−1.90	−0.89	**−7.02**
	Average	**−16.16**	**−10.50**	**−5.46**	**−1.61**	**−0.01**	
Panel B: IVA Allocation Difference, IVA Only							
		0	**25**	**50**	**75**	**100**	**Average**
AIR (%)	**3.0**	−9.60	−7.42	−2.01	−0.47	0.36	**−3.83**
	3.5	−13.06	−5.79	−3.90	−0.38	0.12	**−4.60**
	4.0	−12.62	−8.05	−3.18	0.12	0.00	**−4.75**
	4.5	−11.67	−5.66	−2.58	−0.05	0.00	**−3.99**
	5.0	−12.75	−5.29	−0.72	−0.21	−0.77	**−3.95**
	Average	**−11.94**	**−6.44**	**−2.48**	**−0.20**	**−0.06**	

Original Analysis Assumption (shaded cells: AIR 4.0, 100% equity)

The results in table 6 clearly suggest that the IVA should be invested as aggressively as possible, because the IVA allocations decrease significantly for more conservative IVA equity allocations (e.g., the average allocation to the IVA is 16.16 percent with a 0-percent equity allocation when both products are available, versus −0.01 percent when the IVA equity allocation is 100 percent). Focusing solely on the 100-percent equity allocation scenarios, allocations appeared to increase slightly for lower AIRs, although the differences are not that significant. This suggests lower AIRs are slightly better, on average, likely because they result in a higher level of income from the IVA (on average) later in retirement, when the probability (and magnitude) of portfolio failure is greater. In reality, though, the optimal AIR is likely a more nuanced decision based on the situation and preferences of the retiree. For example, a retiree with a higher subjective discount rate is likely better off with a higher AIR, and a retiree with a lower subjective discount rate is likely better off with a lower AIR.

The results in table 6 clearly suggest the IVAs should be invested relatively aggressively.[3] In the primary analysis, it was assumed that the annuity purchase decision is independent of the portfolio risk level. In reality, the two decisions should be made jointly. For example, the extent that an IFA is bond-like would suggest a higher allocation to an IFA would be accompanied by a more aggressive portfolio allocation to balance out the overall risk level of the retiree's total wealth.

It's not necessarily clear, though, what the exact impact the IVA purchase decision should have on the portfolio. Although the underlying portfolio may be invested aggressively (e.g., 100-percent equities), the guaranteed lifetime income generated from the annuity may result in an effective risk level of the annuity that is less than the risk level of the portfolio. This is something noted by Xiong et al. (2010) when exploring the effective risk of the variable annuity subaccounts when offered with a GLWB.

To determine the effective risk of these annuities (IFAs and IVAs), an additional analysis was conducted where the optimal equity allocation is determined first using the utility model outlined in appendix 1. For this initial test, annuities are not considered and equity allocations ranging from 0 percent to 100 percent, in 5-percent increments, are tested (given the base assumptions of the scenario). In the original model the equity allocation was treated as an input, but for this analysis it is treated as an output, where it is solved for, holding everything else constant.

Given the optimal equity allocation for the portfolio that does not consider an annuity, next the optimal allocation is re-determined where 9.09 percent of the portfolio is used to purchase an annuity. This purchase level may not be optimal, but by allocating to the annuity and re-estimating the optimal portfolio allocation, it is possible to determine the effective risk of the annuity purchase. The potential change in equity allocations is limited to ±10 percent of the original allocation in 2-percent increments. This limits us to considering only scenarios where the initial equity allocation (without the annuity) was between 10 percent and 90 percent equities.

The change in the optimal equity allocation provides clear guidance on the effective risk of the annuity. For example, if the optimal portfolio equity allocation goes from 50 percent before the annuity purchase to 60 percent after the purchase, the effective risk of the annuity purchased would be relatively bond-like. Equation (2) is used to solve for the effective risk for each of the test scenarios, where the effective equity allocation e_e is a function of the initial equity allocation e_i, the initial total financial assets W_i, the equity allocation after considering the annuity purchase e_a, the remaining wealth after the annuity purchase W_a, and the annuity purchase amount P_{ann}.

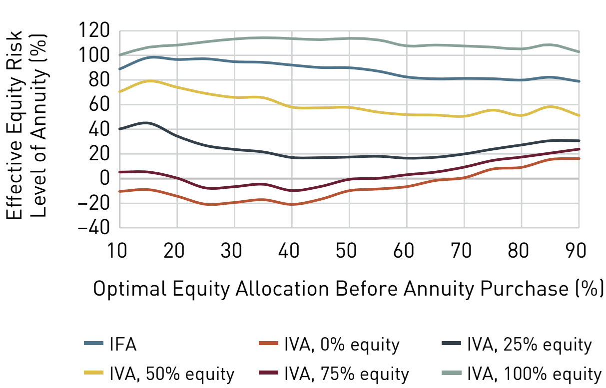
$$e_e = \frac{(e_i \times W_i) - (e_a \times W_a)}{P_{ann}} \qquad (2)$$

Six different annuity purchase-types are considered, IFAs as well as IVAs, with equity allocations of 0 percent, 25 percent, 50 percent, 75 percent, and 100 percent. For each of the six product allocations, 1,000 scenarios were run. The average effective risk level (solved using equation 2) for the respective scenarios, by optimal initial equity allocation, is shown in figure 1.

Figure 1 suggests that the effective risk of the annuities is consistent with the underlying risk of the products. For example, IFAs are, on average, approximately 100-percent bond-like and 0-percent equity-like. This is consistent with expectations given the fixed payment associated with IFAs, which is structurally similar to receiving coupons from a bond. These results suggest that if a retiree wanted to purchase an IFA, the funds should be sourced from the bond portion of the portfolio, which would make the remaining portfolio more aggressive (although the effective risk of the investor's total wealth would be the same). These results are consistent with research by Blanchett and Finke (2018), among others.

Figure 1

THE EFFECTIVE RISK OF IMMEDIATE ANNUITIES



For the IVA, the effective risk also roughly corresponded to the underlying equity allocation of the IVA. This is also consistent with expectations. Given the above analysis, which suggests the optimal equity allocation for an IVA was 100-percent equities, IVA purchases should be made from the equity allocation of the retiree's portfolio (again, in an attempt to keep the effective risk of the investor's portfolio constant after the purchase).

CONCLUSIONS

Every retiree household is different. The greater the range of potential products and solutions a financial advisor has available to recommend to the household, the better the retirement-income strategy is likely to be. This article explores the potential benefits associated with using immediate fixed annuities (IFAs) and immediate variable annuities (IVAs), with a particular focus on when each is the best fit. The analysis suggests there is considerable benefit to incorporating additional guaranteed income into retirement-income strategies, a finding that is consistent with past research. The analysis also suggests ways of helping retirees determine which form of guaranteed income is optimal. Many financial advisors often talk about the benefits of diversification from a portfolio perspective, but the same concepts also apply to retirement-income products.

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ENDNOTES

1. For readers interested in learning more about tontines, see Milevsky (2015).
2. These scenarios can be obtained by contacting the author.
3. Note that this analysis assumes that both products (IFAs and IVAs) are available for purchase. If, for some reason, the client did not have access to an IFA, it could make sense to invest the IVA more conservatively. Pricing considerations also could impact the optimal equity level (e.g., if the payout from the IVA is considerably higher than the payout from the IFA).

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APPENDIX 1: ANNUITY ALLOCATION MODEL

For each simulated income path, the utility-equivalent constant income level is calculated based on the elasticity of intertemporal substitution parameter, which is denoted as II. That is, for a given simulated income path, II is the constant amount of income with the same utility as the actual income path. This is given by

$$II = \left(\frac{\sum_{t=0}^{T} q_t \,(1+\rho)^{-t} I_t^{\frac{\eta-1}{\eta}}}{\sum_{t=0}^{T} q_t \,(1+\rho)^{-t}} \right)^{\frac{\eta}{\eta-1}} \tag{A1.1}$$

Where I_t is the level of income in year t; q_t is the probability of surviving to at least year t, using the Gompertz law of mortality, outlined in appendix 3; T is the last year for which q_t>0; and ρ is the investor's subjective discount rate.

The value of the potential bequest is denoted along path i at time t, B_{it}. Above, the probability of surviving is defined to at least year t as q_t. So, the probability of dying in year t is q_t–q_t+1. These probabilities are used together with the subjective discount rate to calculate a weighted average bequest for each path i:

$$\bar{B}_i = \frac{\sum_{t=0}^{T} (q_t - q_{t+1})(1+\rho)^{-t} B_{it}}{\sum_{t=0}^{T} (q_t - q_{t+1})(1+\rho)^{-t}} \tag{A1.2}$$

$\bar{B}_i$ and II_i are combined to form a measure of the utility of path i in the same units as income. Because II_i is the constant level of income that has the same utility as the actual path of income, it can be expressed as a lump sum (the discounted value of the income stream) at time 0 by multiplying it by

$$\Delta = \sum_{t=0}^{T} q_t (1+\rho)^{-t} \tag{A1.3}$$

Therefore $\bar{B}_i$ can be converted to an equivalent constant level of income by dividing it by Δ. To translate $\bar{B}_i/\Delta$ into the incremental benefit of the possibility of leaving a bequest in addition to the stream of income under path i, the parameter τ is introduced, which measures the strength of the bequest motive. Hence the constant level of income that is equivalent to the income path together with the possible bequests of each year is $II_i + \tau\, \bar{B}_i/\Delta$.

The expected utility is measured using the CRRA utility function with its risk tolerance parameter θ that was introduced in equation 1:

$$EU = \sum_{i=1}^{M} \rho_i \frac{\theta}{\theta-1} \left(II_i + \tau \bar{B}_i/\Delta\right)^{\frac{\theta-1}{\theta}} \tag{A1.4}$$

where M is the number of paths, the subscript i denotes which of M paths is being referred to, and ρ_i is the probability of path i occurring, which is set to $1/M$. Y is defined as the constant value for II that will yield this level of expected utility. This is the certainty-equivalent of the stochastic utility-adjusted income II. Y is given by

$$Y = \left[\sum_{i=1}^{M} \rho_i (\mathrm{II_i} + \tau\, \bar{B}_i/\Delta)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{A1.5}$$

The optimal strategy would be the one that maximizes the value of Y.

For each scenario, the assumed initial Social Security payment is subtracted from the certainty-equivalent of the stochastic utility-adjusted income (Y) to determine value of annuitization for the portfolio monies (P). In order for the household to actually annuitize, the utility generated from immediate annuitization must be greater than or equal to the utility generated from not annuitizing, depending on the household's liquidity preference (l) based on the percentage of the portfolio annuitized. For example, if P=50 without annuitization, and l=10 percent, the utility hurdle for annuitizing 20 percent of the portfolio would be P=51 [(1 + (20 × 10%)) × 50 = 51].

APPENDIX 2: HISTORICAL CORRELATIONS AMONG ASSET CLASSES

	Inflation	Int Govt	Large Stock
IA SBBI US Inflation	1.00		
IA SBBI US IT Gobt TR USD	0.02	1.00	
Ibbotson SBBI US Large Stock TR USD	0.00	−0.03	1.00

APPENDIX 3: GOMPERTZ MORTALITY MODEL

Mortality is modeled using the Gompertz law of mortality, named for Benjamin Gompertz. Gompertz determined that a person's probability of dying increases at a relatively constant exponential rate as age increases. The formulation of the Gompertz law for mortality used here is based on the work of Milevsky (2012), where the probability of survival to age t, conditional on a life at age a, is given by equation A2.1, where m is the modal lifespan and b is the dispersion coefficient.

$$q_t = \exp\{\exp\{\frac{a-m}{b}\}(1-\exp\{\frac{t-a}{b}\})\} \qquad \text{(A2.1)}$$

Gompertz parameters are fitted to the Society of Actuaries 2012 Immediate Annuity Mortality Table with improvement to 2019 by minimizing the sum of squared differences from the test parameters and the actual mortality estimates in the table at age sixty-five for a male and female. The model lifespan is 90.47 and 92.46 for male and female, respectively, and the dispersion coefficient is 8.53 and 8.33 for male and female, respectively.

APPENDIX 4: DYNAMIC WITHDRAWAL MODEL

The discretionary withdrawal amount is determined using a dynamic withdrawal rule that is similar to the approach used to calculate required minimum distributions (RMDs) from qualified accounts (e.g., individual retirement accounts). For the first step, the median life expectancy of the retiree household (LE) is estimated at each age for the length of retirement. Next, one is divided by that value (LE), which is referred to as the base RMD factor, and that is multiplied by the amount of need that is deemed to be flexible (or discretionary) in the first year of retirement. That percentage is adjusted so the total flexible withdrawal is equal to the first-year need. That same adjustment factor is then applied to the future one over LE withdrawal rates, by year. For example, let's assume a portfolio value of $1,000, an initial total retirement need of $50 where 50 percent is flexible, and median life expectancy of thirty years. Given the median life expectancy of thirty years, the base RMD factor would be 3.33 percent (1 ÷ 30 = 3.33%). Given the flexible income target of $25 ($50 × 50% = $25) and flexible initial balance of $500 ($1,000 × 50%), the adjustment factor would need to be 1.5 so that the initial flexible withdrawal rate equaled the target amount [(1.5 ÷ 30) × $500 = $25]. This adjustment factor (1.5) would be used with all subsequent RMD factors and applied to the initial balance to determine the flexible withdrawal amount for the respective year.

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