



# **Gear Validation with Multiple Tooth Contact**

How the Digital Twin Benefits Gear Systems and Gear  
Manufacturing

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# Gear Validation with Multiple Tooth Contact

## Abstract:

3DCS Variation Monte Carlo analysis with multiple tooth contacts and gear rotations are used on two standard gear tests, axial backlash, angle backlash and a flank test pattern. Charts, illustrations and results for a Spur Gear Train are used to present the conclusions graphically.

Axial and Angular Backlash are calculated for the gears and charted by angle degree. Additionally, the flank test results with multiple tooth to tooth contact are also shown. The Angle Backlash "Kickback" analysis for the one gear set can be derived from the calculation. "Kickback" is calculated as the maximum range of the angle backlash.

The two standard tests compare a gear with specified GD&T for the bore and teeth against a Nominal Gear, Master Gear. The standard test are:

- $F_i$  Total Radial Composite Deviation - variation in center distance rotated 1 revolution. This is the Axial Backlash Range.
- $f_{tp}$  - Single Pitch Deviation - deviation of the arc length between adjacent teeth along the reference circle, compared to nominal.

The analysis model statistically summarizes the Monte Carlo sequence stages for each measurement to identify the GD&T frames and hole/pin clearances for the contribution percentage. In the model, rotational capability and the analysis of rotation with three teeth in potential contact are used for collision detection. The model also uses visual animation to provide important clues and verifications by way of frame by frame move sequences. These identify specific random deviation sequences and allow for tolerance isolation.

One finding is that the range of angular and axial backlash measurements of a gear train can be analyzed with a single tooth to tooth rotation of the Output Gear. That is, for a single stage gear train, the Monte Carlo evaluation does not require a 360° rotation of the Output Gear, only the angle discretization for one tooth to tooth rotation of the Output Gear.

## Introduction:

Dimensional analysis, using multiple tooth contacts and rotation, can identify opportunities to meet the design's dimensional objectives and cost targets early in the design process. These refinements may include the gear, gear tooth, bearing or housing specifications, machining refinements, build sequence changes, part consolidations, and gear grade modifications. This paper focuses on the gear and gear tooth effects. [2]

3DCS Variation Analyst software uses Simulation stages where additional measurements can be added and evaluated. In the Nominal Stage, all the tolerances and GD&T are set to nominal

values. The following stage, Deviate, where a Monte Carlo Method sets each tolerance to a random value in a specific sequence. Run Analysis statistically summarizes the Monte Carlo sequence stages for each measurement. The Contributor GD&T frames and hole/pin clearances are identified along with the contribution percentage. Dimensional Engineers use this data to refine the GD&T to better meet the dimensional target. In general, the analysis uses a fixed rotation position of the gears.

Nominal Stage measurements are the baseline for the nominal geometry. Monte Carlo summary measurements calculate the effect on the geometric features with GD&T. These can be compared to the gear design system specifications. [4]

The analysis model utilized Multi-Tooth contact and gear rotation capabilities. 3DCS Mechanical Modeler Add-On was used to rotate the gears and track the measurement angles. Each angle measurement result is placed in a spreadsheet. A Monte Carlo simulation is performed for each angle and statistics are displayed by angle. In this case, the gears are rotated from the initial position with multiple consecutive teeth in contact. The angle of rotation affects the measurements, so each gear angle is a different mean shift.

Axial and Angular Backlash, [1] GEAR BACKLASH, are included in the DCS Gear Module. Single Pitch Deviation and Total Radial Composite Deviation, [1] ACCURACY OF GEARS, use existing DCS capability and Axial Backlash respectively.

Figure 1 and Figure 2 below show the Axial Backlash contacts. In axial backlash, the Green Gear is allowed to rotate and center on the Blue Gear, then the distance to contact is measured.

Figure 3 illustrates Single Pitch Deviation ( $f_{tp}$ ) - The deviation between the actual measured pitch value between any adjacent tooth surface and the theoretical circular pitch.

In the mechanical testing,  $F_i''$ , Total Radial Composite Deviation measures the variation in center distance rotated in one  $360^\circ$  revolution. In the Monte Carlo simulation the variation in center distance, for a rotation of one tooth is shown in Figure 4. Axial Backlash is equivalent to  $F_i''$  using the Monte Carlo method.

The Standard Tests are calculated from the spreadsheet and the results are also plotted in the graph interpretation. The rotation angle is used as the ordinate axis in the graphs for the deviation calculation. Each angle of the test uses a Monte Carlo Analysis of a specific number of runs to calculate the test results.

The total angle of the tooth rotation is calculated as follows: 360 degrees divided by the number of teeth. [1] GEAR TRAINS. This constitutes the total rotation of one tooth. In Monte Carlo simulation, this tooth is used to simulate the entire rotation of the gear. A gear with 12 teeth requires 12 runs to simulate a complete rotation. The next 12 runs would simulate a new gear, 1200 runs simulates 100 gears. In an actual gear, the same bore perpendicularity pore position would be used throughout the rotation, whereas in Monte Carlo simulations, these are always a

random selection for each of the 12 runs. The statistical results for the standard tests are not significantly affected by these differences.

One specific rotation angle can be analyzed in a separate 3DCS Monte Carlo analysis to calculate more information on the GD&T effect. This is calculated in the DCS Run Analysis results platform and includes Contribution Analysis. The percentage of the variation by features and tolerances is available as well as a histogram of the measurement variation distribution of the runs. In addition, min and max ranges as well as calculated values based on the distribution.

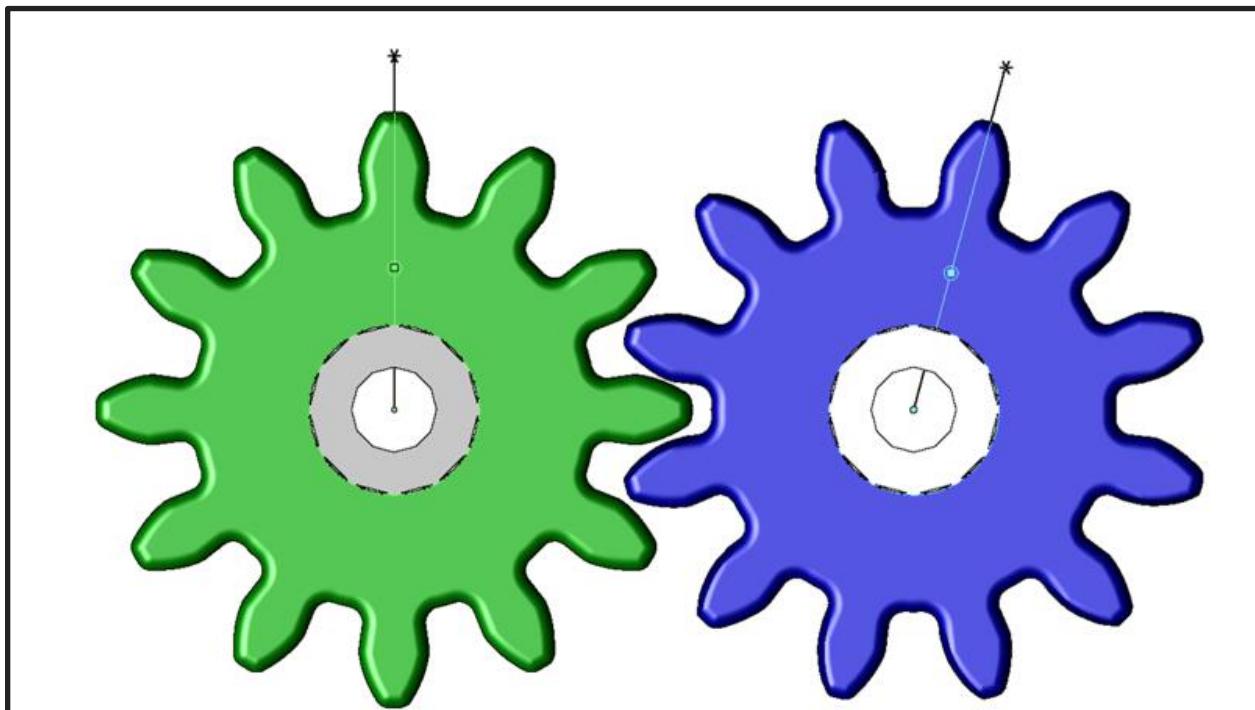


Figure 1. The initial position of the Gears in Axial Backlash,  $F_i$ , with the contact points. The green Drive Gear is at Top Dead Center, TDC or  $0^\circ$  degrees.

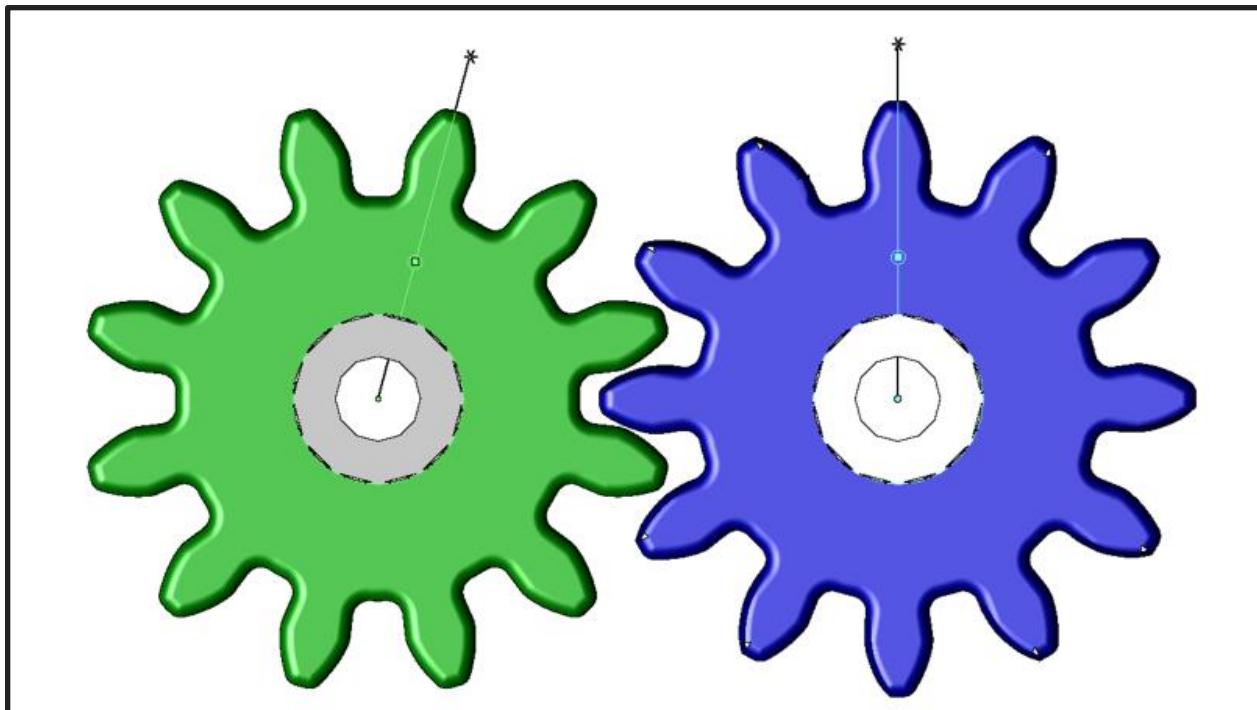


Figure 2. A 15° rotated position of the green Drive Gear with the Multi Tooth, Rotational Contacts for Axial Backlash and  $Fi''$ .

A visual display of the components at the Nominal, Minimum and Maximum tolerance values for a specific contributor are available. All measurements can be reviewed on this display. Since The GD&T tolerances can be measured as well, the tolerance values can be determined for the Maximum or Minimum deviation of a measurement. This rich analysis capability is used to quickly identify, refine and validate dimensional modifications for conformance and reduced cost, a key factor to meet the Gear Train Objectives.

In addition to contributor analysis, Mean Shift analysis is used to identify process changes, such as changes in tolerance ranges or shims. These may validate the practical changes using the design team's suggestions. For instance, a shim may change the mounting distance and the clearance between the gear teeth, changing the angular backlash.

Each angle in the standard measurements is also mean shift analysis and is analyzed with the angle as the factor in the spreadsheet. The simulated tooth contact pattern is used to calculate  $Fi''$  and  $f_{tp}$  in the 3DCS Mechanical spreadsheet for the different angles. The Maximum and Minimum values are used in the calculation. The Monte Carlo results include the eccentricity of the bore, [4].

The contact pattern is plotted for visual evaluation as well. The standard tests,  $Fi''$  and  $f_{tp}$  are part of the gear specification to ensure that gear noise, gear vibration and life expectancy meet the product specification.

The variation analysis of one angle uses the same number of runs and allows a deeper dive into the statistical analysis including the Variation Contribution of individual teeth.

## Gear Specifications

The accurate representation of the gear teeth geometry relative to the measurement and specification requirements is the key to gear dimensional analysis. Gear teeth specifications can be combined into a composite tolerance which is convenient for the analysis tolerances for contact tolerances. Some tolerances may be useful to apply separately to each feature/surface, such as the gear center bore and tooth profile tolerances to define the GD&T of the gear teeth.

Tooth contact measurements are made from the initial drive contact, through pitch position and the release from contact. These are made in discrete equal angle rotation of the gears. The distance between gear centers, axial backlash, angular backlash and single pitch deviation are measured.

The standard tests are from a Monte Carlo simulation using the methods described in [1] ACCURACY OF GEARS.

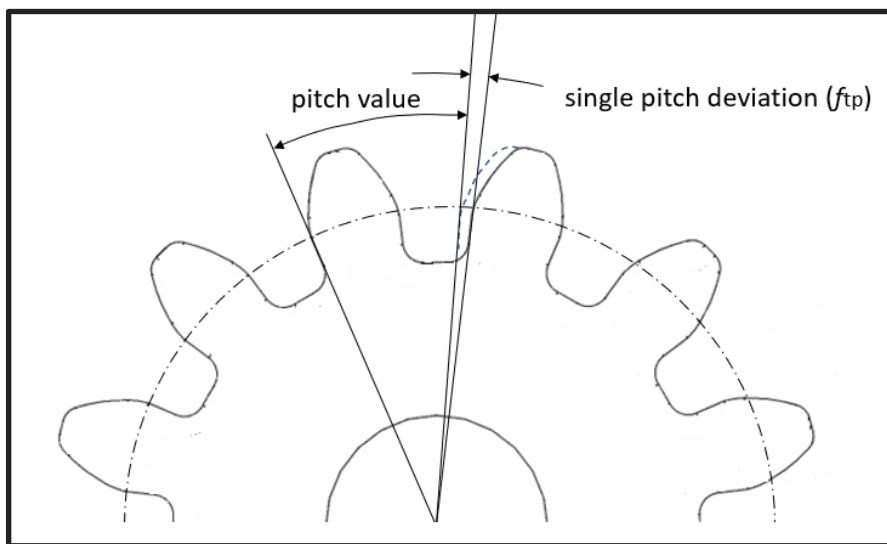


Figure 3. Single Pitch Deviation ( $f_{tp}$ ) - The deviation between the actual measured pitch value between any adjacent tooth surface and the theoretical circular pitch.

In Total Radial Composite Deviation measurements,  $F_i''$ , the green Nominal Gear, Master Gear, is rotated and the blue gear, Work Gear, is pressed against it. See Figure 4. An axial reading is used to measure the distance between the gears with two points of contact. Mechanical testing is done on a Gear Rolling Gauge also called a Double Flank Gear Gauge. The resulting

measurement,  $Fi''$ , measures the variation in center distance rotated in one  $360^\circ$  revolution. In the Monte Carlo simulation the variation in center distance, for a rotation of one tooth.

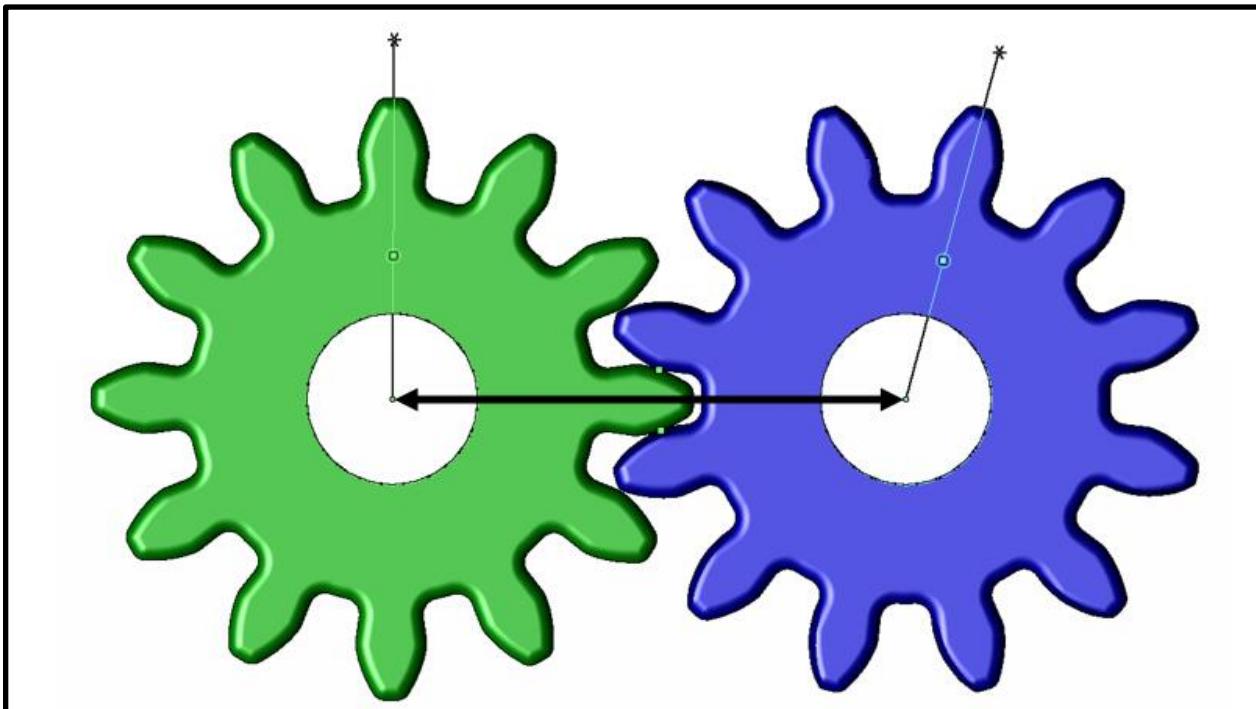


Figure 4. Standard Measurements  $Fi''$ : using the Monte Carlo Method for the Distance between Gear Centers.  $Fi''$  is the maximum distance minus the minimum distance for a  $360^\circ$  rotation under pressure to push the gears together. The two contact points indicate that this is a double flank test.

## Geometric, GD&T, Monte Carlo and Measurement Models:

3DCS Multi-Cad is used to model the gear. The configuration for the  $f_{tp}$  and  $Fi$  is shown, Figures 3 and 4. The green gear is the Nominal Gear, Master Gear, and is at nominal tolerance. The bearings for both gears are set at nominal. The 3DCS Gear Module is used to create the basis for simulation once the gears are defined.

## Geometric

The Gear CAD master data is a Solid Model accurately representing the nominal GD&T. Contact Mesh is made using the Standards of the Gear Module. A spur gear is the focus of the paper.

Gear	Number of Teeth	Modulus	Pressure Angle
<b>Drive &amp; Output pinion</b>	<b>12</b>	<b>3.0</b>	<b>20°</b>

Table 1. Gear Specifications for the standard test.

## GD&T

The Toleranced Gear bore is Datum A. While the gear face is not a Datum, it has both position and perpendicular tolerances. The bore points used in the Gear Module are bore feature points, which inherit the tolerances placed on the bore surface.

As the gear is moved to the bearing, the bore is aligned to the outer race in the primary move. Both the Perpendicularity and Positional tolerance of the tolerances gear are active. In the secondary move, the bore points are now aligned with the bearing outer race.

The contacting gear teeth each have the same profile tolerance.

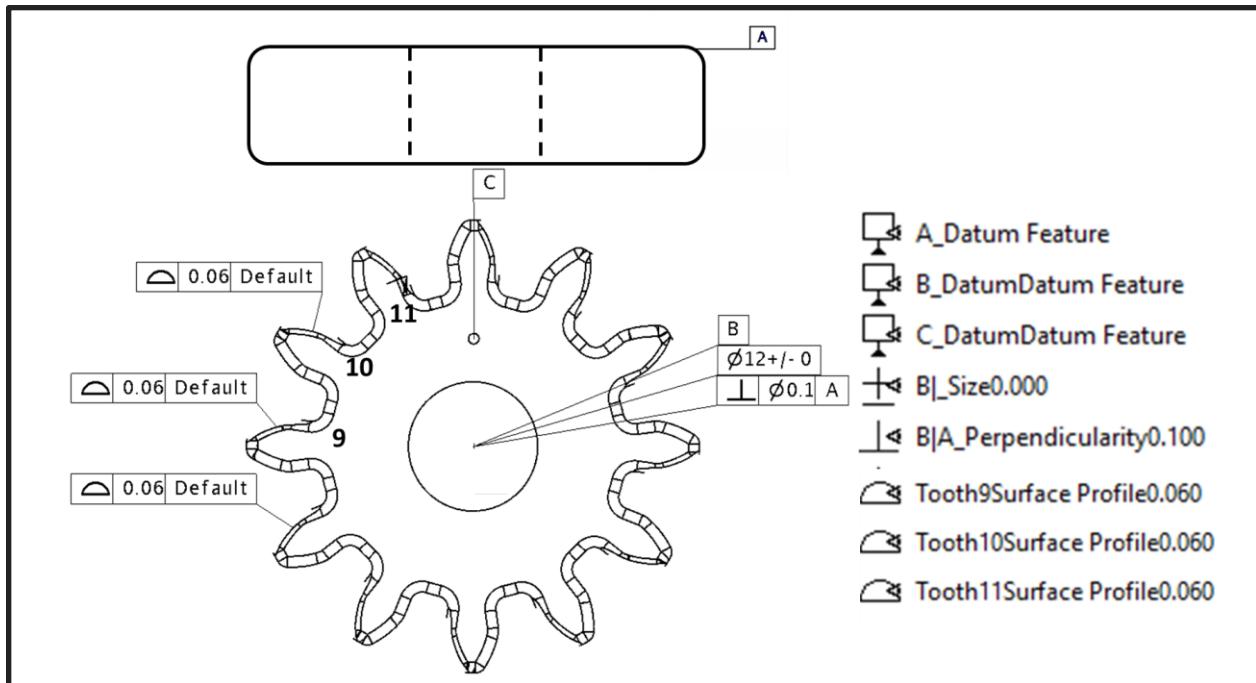


Figure 5. GD&T for the Toleranced Gear.

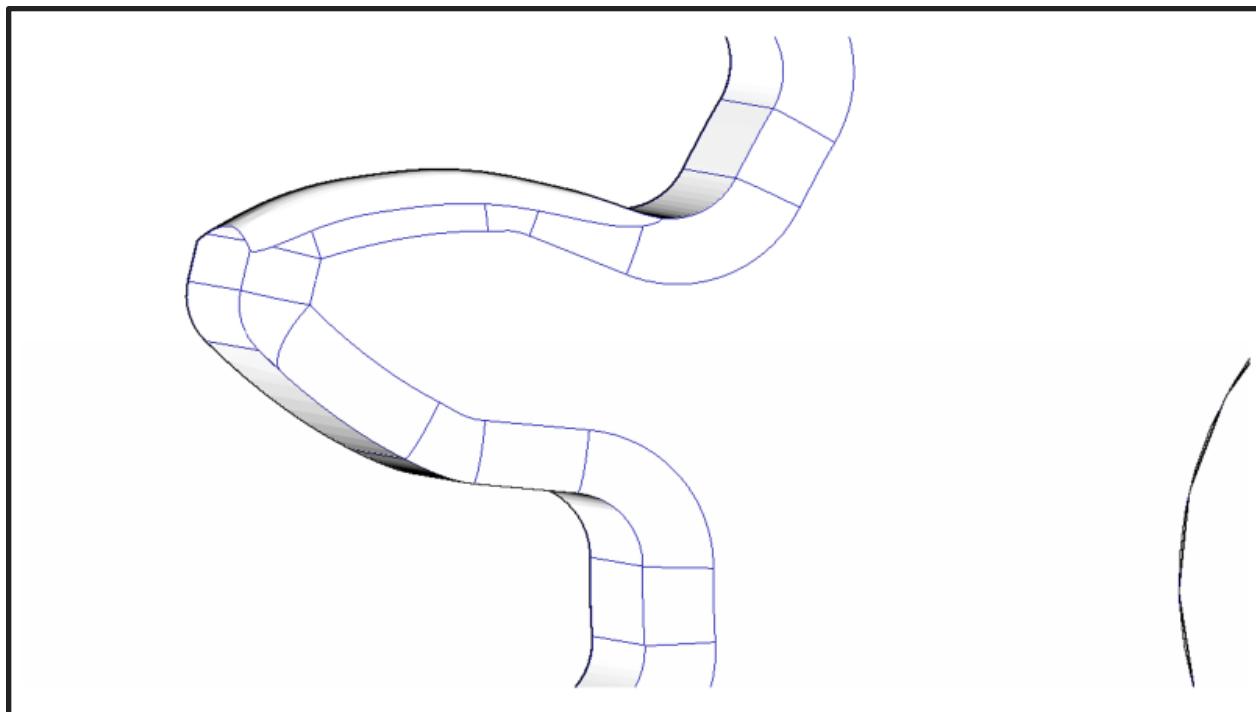


Figure 6. Tooth Crown for the Toleranced Gear. The drive side, above, of the top gear is crowned. The Nominal Gear is not crowned.

## Geometrical Behavior Simulation:

The Gears initial position is Top Dead Center, TDC, in assembly position. The Nominal Gear starts at  $0^\circ$  and is rotated  $30^\circ$  or is  $360^\circ \div 12^\circ$  one full tooth rotation. A full rotation can be simulated by rotating the teeth  $30^\circ$ , one tooth to tooth rotation. All teeth are dimensioned identically and are independent of each other, so one tooth rotation variation is the same as another.

## Rotational Monte Carlo Measurement Analysis

The teeth are rotated using 3DCS Mechanical rotation in one degree increments, and plotted below. The Mounting Distance is calculated at the Nominal, Minimum and Maximum values. These are saved in a spreadsheet used to calculate the Maximum Tooth-to-Tooth Total Radial Composite Deviation. The measurements are plotted using the spreadsheet.

As the gear rotates, the contact locations transition to the next adjacent teeth. In this case, the contact region was expanded into additional teeth in the DCS Gear Module.

DCS Mechanical Results are provided by an Excel Spreadsheet. Excel was used to extract the charts and calculations.

The Angular Backlash is calculated with a Mounting Distance of 36 mm. The reference circle contact distance is 36 mm. The Tooth Normal Backlash subtracts material from the tooth to prevent gear locking.

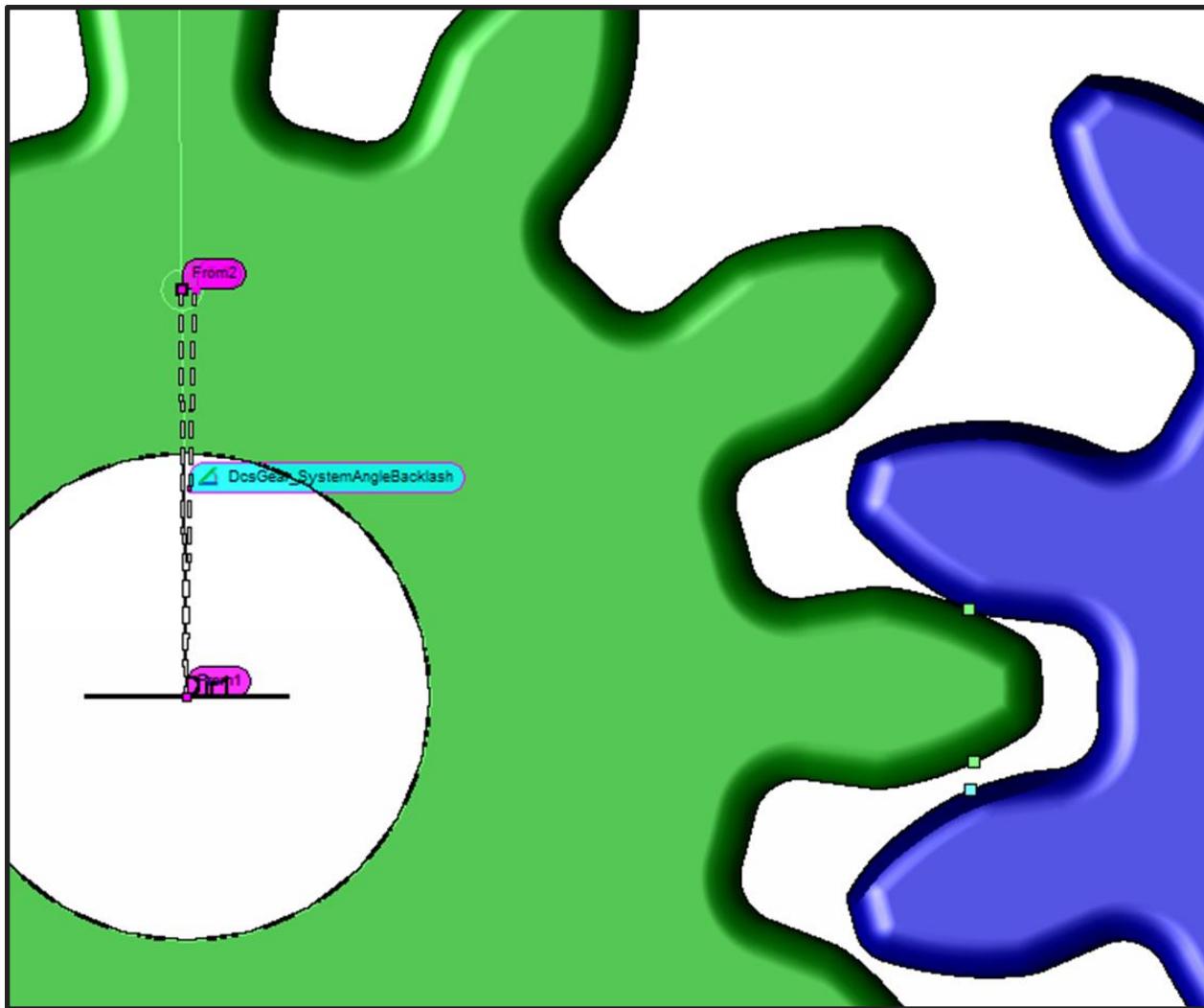


Figure 7. The angular backlash measures the amount of angular clearance between the Nominal Gear teeth and the Tolerance Gear teeth. The Drive direction is Clockwise for the green gear. The bearings are at a fixed distance apart. The angular backlash measures the angle of clearance between the Nominal Gear teeth and the Tolerance Gear teeth.

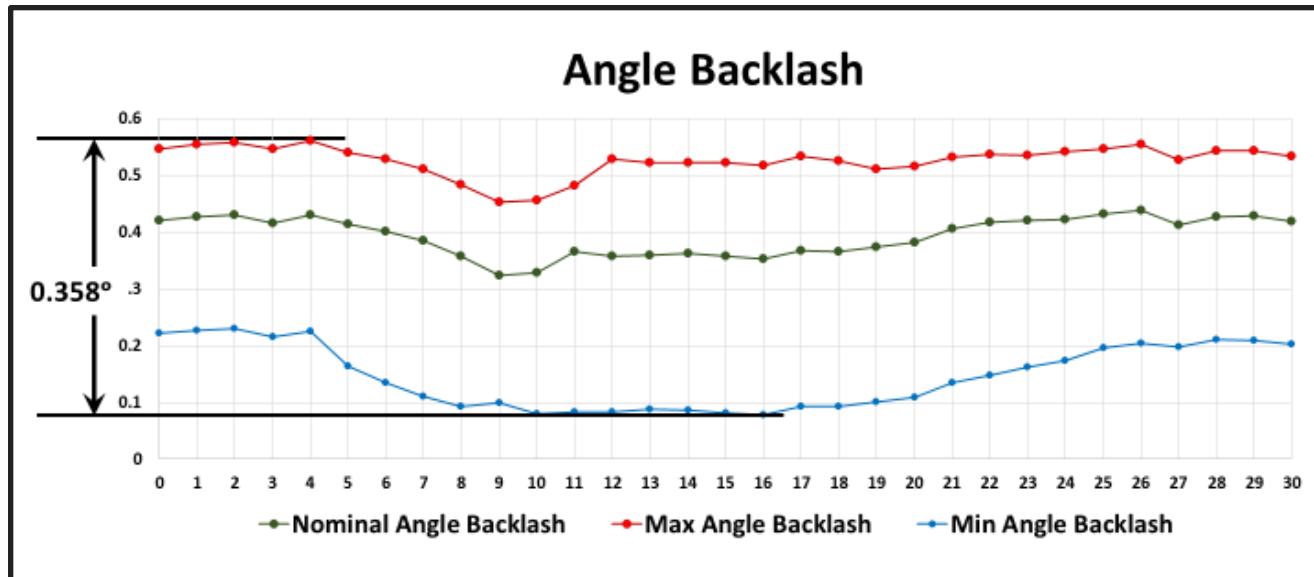


Figure 8. Angular Backlash range, also known as Kickback, is  $0.358^\circ$

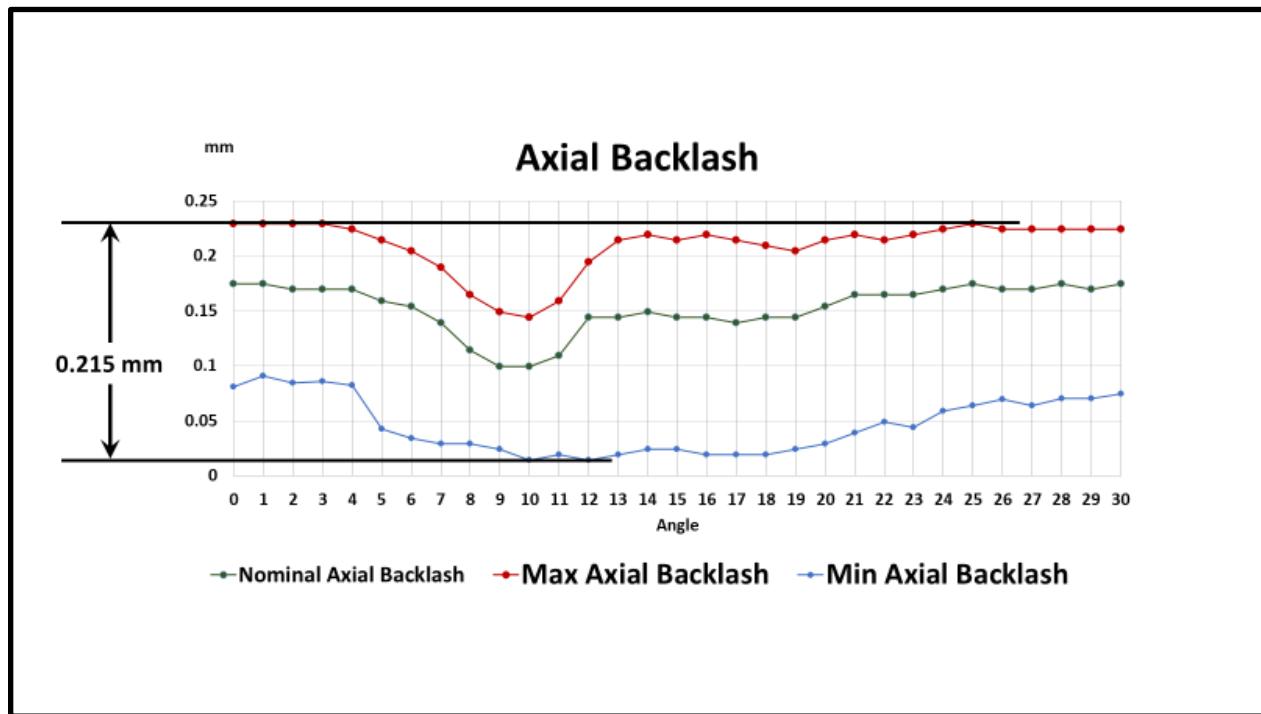


Figure 9. Axial Backlash, a.k.a., Total Radial Composite Deviation, a.k.a,  $F_i$ ", allows the Drive Gear to rotate during contact. The drive gear is rotated so that both the drive and reverse make contact.

The flank test is used to track the contact points and ensure that they are within tolerance of the contact target on the flank of the tooth. Both visual confirmation and measurements can be used. The visual confirmation is shown in this paper.

The contact points for the Drive contact show the gears operating in highest wear condition, usually. The contacts may involve multiple teeth. A reverse contacts are also shown with multiple tooth contacts.

As the gears rotate, the Axial. Backlash contact points are shown where the measurements are made. This is also used for the Total Radial Composite Deviation. This shows the contact locations as the Master Gear is rotated and the Tolerances Gear is pressed and rotated against it.

## Tooth to Tooth Contact Pattern

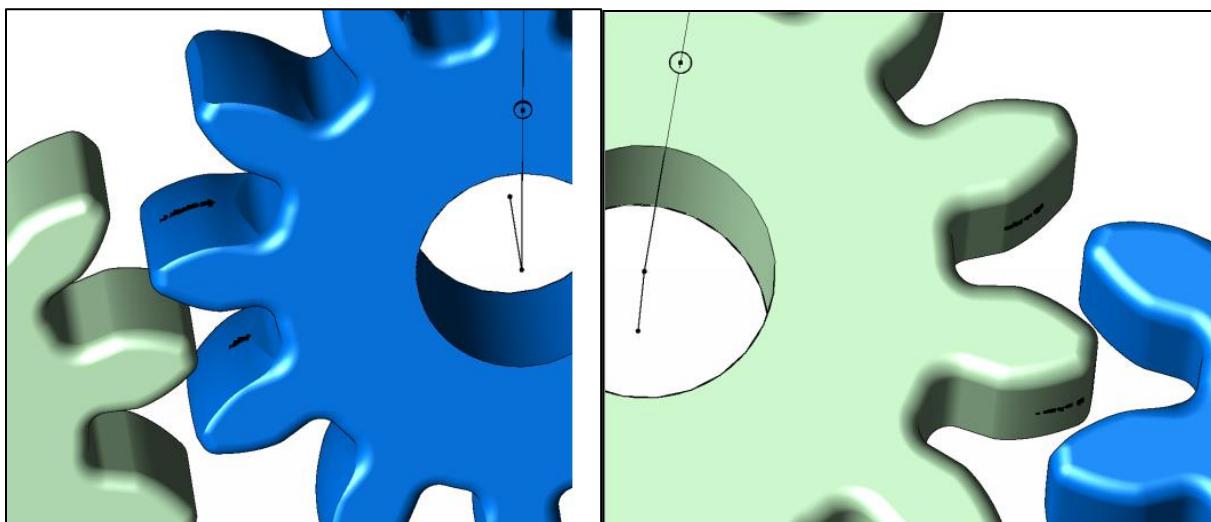


Figure 10. Angle Backlash Drive Flank Contact

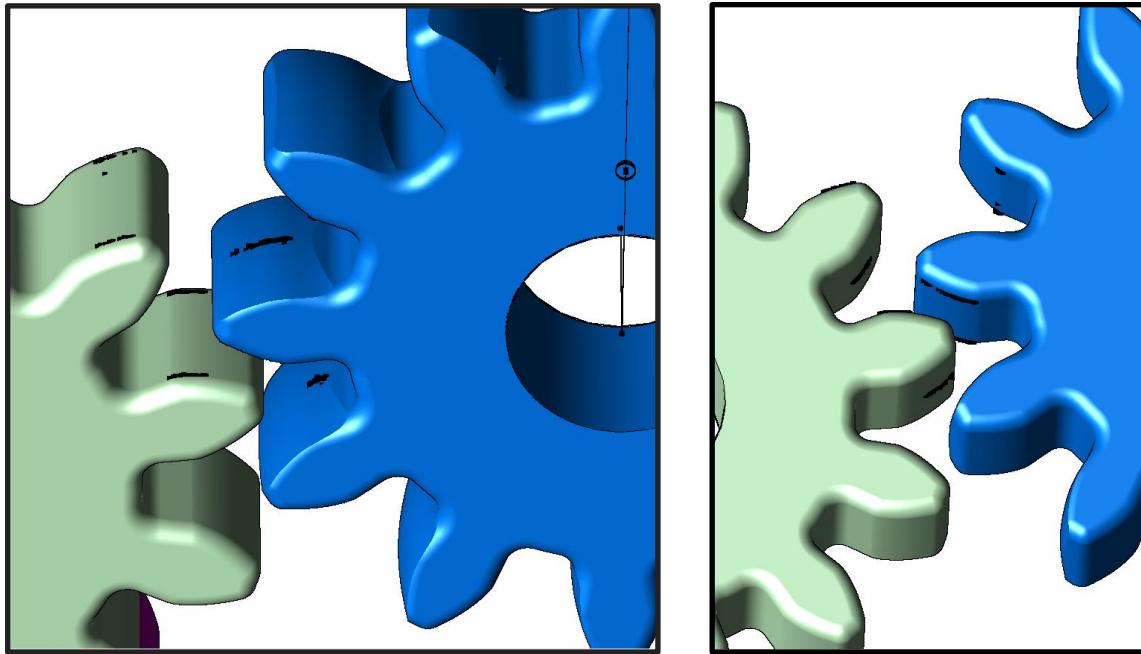


Figure 11. Multiple tooth contact points  $F_i$  runs using the 3DCS Axial Backlash with added flank test. The green Nominal Gear is rotated CW with the Blue Toleranced Gear in constant pressurized contact. The crowned gear in the drive direction has the contact centered, while the reverse direction uncrowned contacts are split toward the edges.

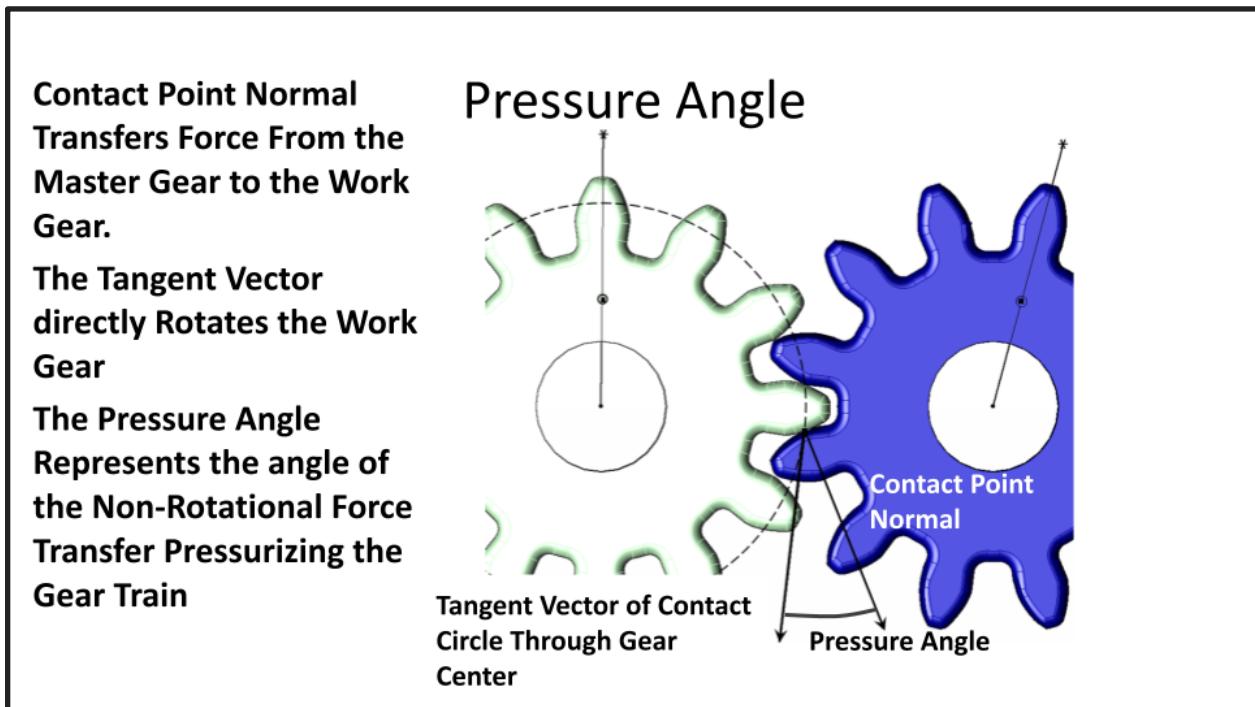


Figure 12. Pressure Angle corresponds to the non-rotational tooth contact force.

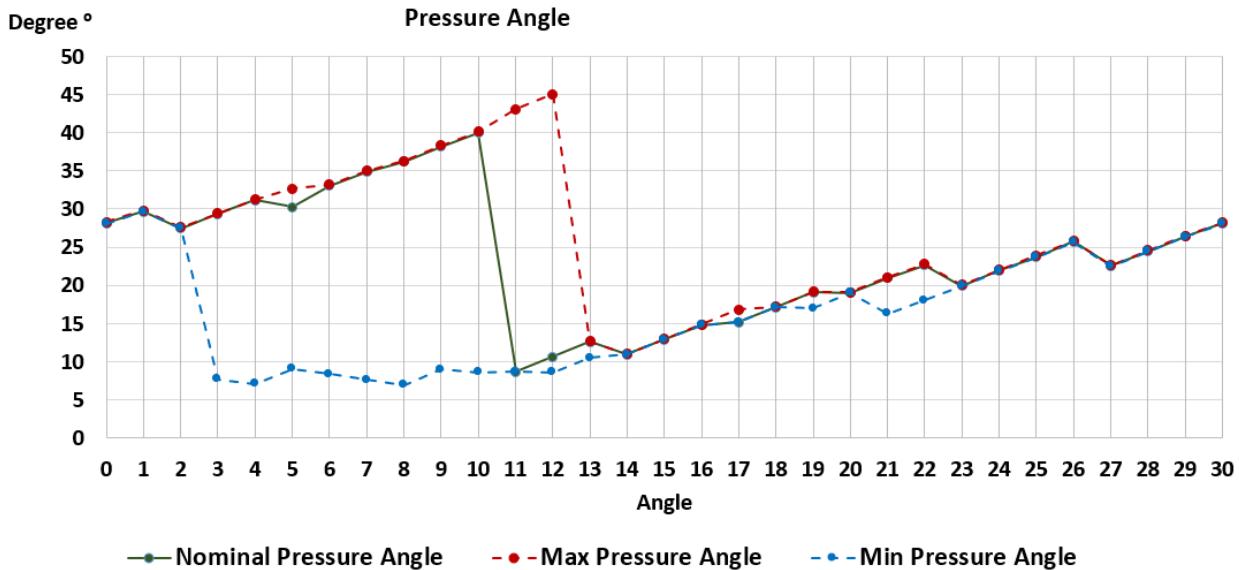


Figure 13. Pressure Angle changes throughout the multi tooth contact region.

The vector between the tip of the tangent vector and the tip of the pressure angle, is the direction and magnitude of a force pushing the gears apart. The force transmits through the gears to the bearings, bearings to the shaft and shaft to the housing. This pressurizes the entire assembly, until the assembly pressure matches the pressure angle.

The pressure angle changes as the gears rotate and is a component of the transmission error. This may cause the gear teeth, gears and gear train to vibrate and / or produce noise which is out of specification.

## Measurement Contribution Analysis:

The Contribution Analysis provides the measurements for a single angle of Axial Backlash. If the angle is coincident with an increment angle and the number of runs is the same, then the measurements will be exactly the same. In addition, the percentage of contribution of the tolerances and a histogram distribution of the measurement is also provided, Figures 14 -16.

The number of runs can be increased for a specific angle to increase the accuracy of the maximum and minimum values. This is a more efficient method for a specific angle, rather than increasing the number of runs in the rotational for all the angle increments.

Axial Backlash Monte Carlo Analysis for 7200 Run using random angles to find the Min and Max values and the angles of the Min Max Values. The Contributors to Variation are shown by inputting the angles where the Max or Min values are obtained and are shown in Figure 15. Teeth in contact can be seen in the image, corresponding to the information in Figure 15.

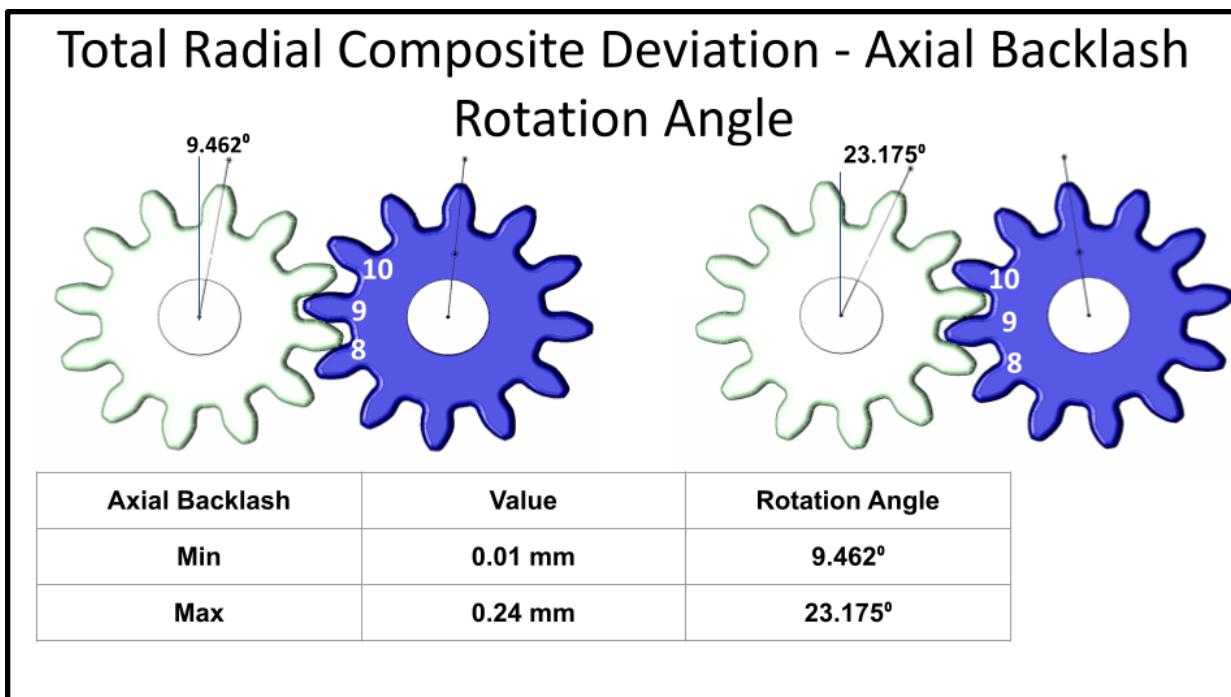


Figure 14. The Toleranced Teeth in contact as the gears rotate. The Min and Max values if the Total Radial Composite Deviation are obtained from the Run Analysis results.

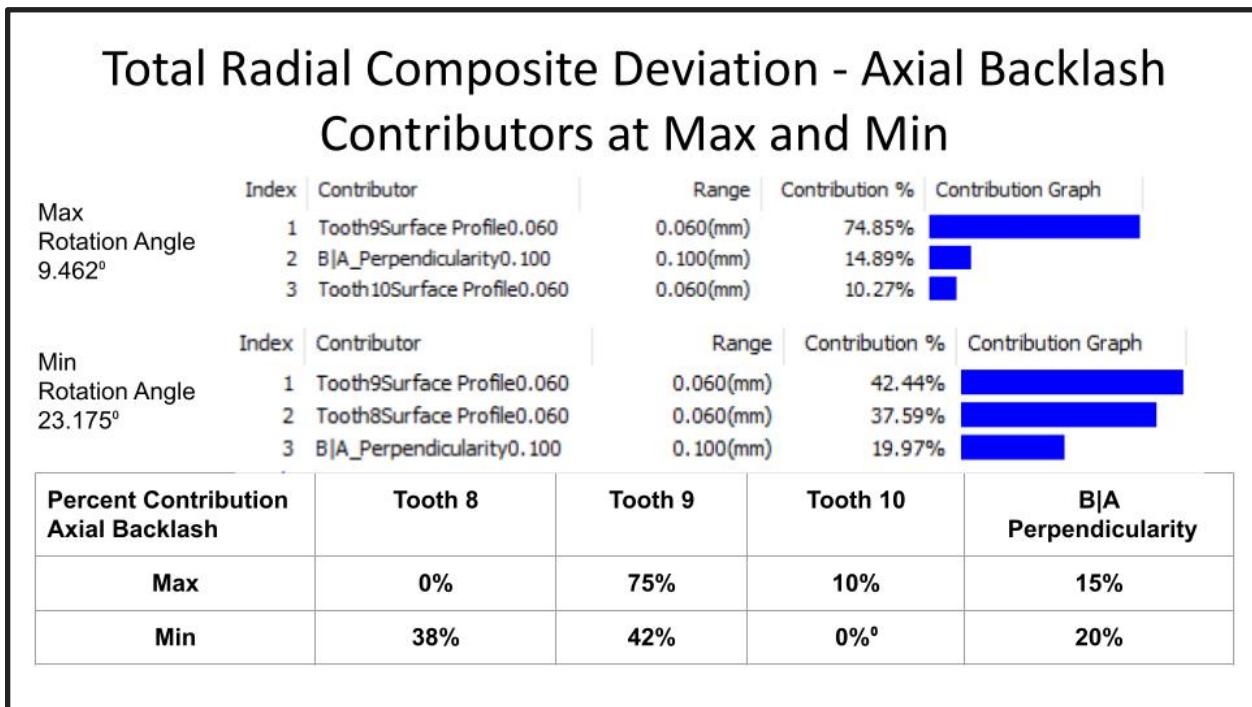


Figure 15. The Contributors to Variation are shown by inputting the angles where the Max or Min values are obtained and using the Run Analysis at each angle.

The single pitch histogram shows the range of the deviation from tooth to tooth measured at the reference circle.

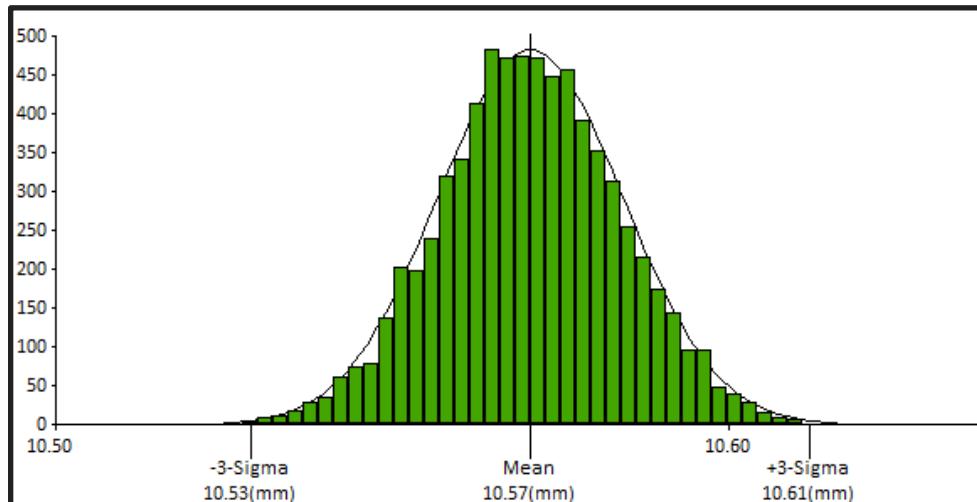


Figure 16. Single Pitch Deviation Histogram from the Measurement Report. The Maximum Value is 10.62 mm, The Minimum Value is 10.52 mm.

## Accuracy of Measurements

The calculation accuracy has 3 components; 1.) Increasing Number# of Runs, 2.) Increasing the discretization of the angle increment and 3.) Increase number of facets representing the CAD model

The mesh accuracy for the tooth. This defines the distance between the faceted model and the spline surfaces. More angle increments will also increase the fidelity of the range of values. In addition, the number of runs will increase the confidence level of calculating the maximum and minimum value.

## Conclusion:

The standard gear test measurements of: 1).  $F_i$ " Total Radial Composite Deviation - variation in center distance rotated 1 revolution; 2).  $f_{ip}$  - Single Pitch Deviation - deviation of the arc length between adjacent teeth along the reference circle, compared to nominal, were calculated and charted. Angle Backlash and Axial Backlash were also included in the measurements and charts. The Monte Carlo simulation of using: 1). Multiple tooth contact; 2). Synchronized gear rotations; 3). GD&T for the gears and gear teeth were calculated using the modified 3DCS Gear Module and standard spreadsheet calculations.

The range of these measurements of a single stage gear train can be analyzed by one tooth to tooth rotation of the Drive Gear. As the Drive Gear rotates one tooth to tooth rotation, the gears in the gear train rotate one tooth to tooth rotation. This is sufficient to analyze a single stage gear train. The Monte Carlo evaluation simulates the measurements 360° rotation of the Drive and Output gears. The angle increments of the one tooth to tooth rotation of the Drive Gear are plotted from the DCS Mechanical generated spreadsheet.

This means the entire gear drive train range of measurement can be obtained by rotating the Drive Gear from Top Dead Center to 360° ÷ Number of Teeth. The rest of the rotations can be obtained by using the Gear Ratios of the Gear Train, [2] Gear Train Chapter.

The Angle Backlash “Kickback” analysis for the one gear set can be derived from the calculation. “Kickback is calculated as the maximum range of the angle backlash.

There are three main methods of evaluation: 1) use Mechanical and review the spread sheet, 2) use Run Analysis and vary the angle randomly, 3) use Run Analysis and set the angle.

Option 1) allows the tracking of the results by angle. Option 2) is used to find the Max Min values and corresponding angles. Option 3) has the contribution level and histogram for that angle.

Single Pitch Deviation,  $f_{tp}$ , is calculated and with the contributor limited to the tooth profile. The Monte Carlo simulation gives the predicted value.

The method outlined above may be expanded to include a single stage gear drive train with multiple gears. This includes the bearing with deviations, housings and other gear types.

## Getting Started with Gears and 3DCS

There are two ways to get started with Gear Digital Twins at DCS:

1. Contact DCS to get a project outlined to get your team hands on experience and get a model into your hands quickly  
<https://www.3dcs.com/technical-support/request>
2. Contact your sales account manager (for current 3DCS users) to get 3DCS Mechanical Modeler Add-on included in your software configuration. Training is highly recommended, as Gear Modeling is a form of Advanced Modeling Technique similar to Finite Element Analysis  
<https://mkt.3dcs.com/contact-us>

## Appreciation

The author has appreciation for the many contributions and technical support of the DCS Team that participated in the development of the DCS Mechanical Gear Module.

Gary Bell  
Dave Johnson  
Brenda Quinlan  
Ben Reese  
Sylvia Rissell  
Spencer Strouse  
Vishnu Priya Vijayakumar  
Mike Ulciny  
Zesheng Zhang  
Ying Qing Zhou

## Preparing the Model

Reverse and Axial can be selected using multiple teeth for the drive, reverse and axial surfaces. These should be compatible with the Counterclockwise selected direction. All move and measurement operations for Axial and Angle Backlash work per usual and measurements, such as Pressure Angle work as the standard model. Flank Test also works.

For the standard test, the Gear Module was used for a two gear model. The Gear Pairs (NomGear and TolGear) were selected for the teeth used in the model. Teeth for the Toleranced Gear are: Tooth8, Tooth9 and Tooth10. The DCS Gear Module is modified to use gear ratios to track the toleranced gear.

In this instance, Drive was selected for the three Blue Gear Toleranced Teeth and the corresponding teeth in the Green Nominal Gear. All the surfaces normally added to Reverse and Axial were also added to Drive. For the multiple teeth.

3DCS Mechanical is set to rotate the Nominal Gear a specified number of degrees, in this case 30 degrees. The angle is measured using the standard angle measurement of the clocking point. The Toleranced Gear is rotated in the opposite direction with the same magnitude.

## References:

[1] KHK Gear Knowledge  
Kohara Gear Industry Co.,Ltd.  
13-17 Nakacho Kawaguchi-shi Saitama-ken, 332-0022, Japan

[2] Managing the Cost of Quality Donald Jasurda, DCS Vice President of Sales

[3] Towards an Improved AGMA Accuracy Classification System on Double-Flank Composite Measurements E. Reiter, Gear Technology, June/July 2012

[4] Statistical tolerance analysis of bevel gear by tooth contact analysis and Monte Carlo simulation Je`ro^me Bruye`re, Industriel et de Production Me`canique, E.N.S.A.M. de Metz, 4 rue A. Fresnel, 57070 Metz Cedex, France, 29 December 2, 2021