Automation in Construction 103 (2019) 300–314

Contents lists available at ScienceDirect

Automation in Construction

journal homepage: www.elsevier.com/locate/autcon

Monte Carlo simulation for tolerance analysis in prefabrication and offsite construction

Christopher Rausch[a,*], Mohammad Nahangi[b], Carl Haas[a], Wanhua Liang[a]

[a] *Department of Civil and Environmental Engineering, University of Waterloo, Canada*
[b] *Department of Civil and Mineral Engineering, University of Toronto, Canada*

ARTICLE INFO

Keywords:
Prefabrication
Offsite construction
Modularisation
Dimensional variability
Tolerance analysis
Rework
Risk management
Monte Carlo simulation

ABSTRACT

Producing assemblies that experience little to no rework during assembly fit-up is a perennial challenge in offsite construction. While the traditional response has been to solve geometric issues onsite at the expense of large rework costs, manufacturing optimisation techniques can mitigate these issues upstream. Tolerance analysis is one such method that is widely used in manufacturing to predict if problems will occur from the accumulation of tolerances and dimensional variability. This article demonstrates Monte Carlo tolerance simulation on a prefabricated construction assembly, where variations up to 37 mm are identified compared to as-built deviations which range up to 30 mm. Process optimisation is also explored, where risk of rework related to dimensional variability is reduced by 65.6% through selection of alternate fabrication processes. To compare the Monte Carlo method to traditional analysis methods, a simplified 1-D tolerance analysis is used. Compared to an as-built deviation of roughly 11 mm, the Monte Carlo method produces a conservative value of 15.4 mm, while other traditional methods are either overly conservative (worst-case tolerance chain has a deviation of 19.8 mm), or overly ambitious (root sum square tolerance chain has a deviation of 4.6 mm). Tolerance analysis through Monte Carlo simulation is shown to be a proactive design tool with several key advantages for prefabricated and offsite construction. First, complex three-dimensional geometric interactions can be readily modelled using very basic tolerance configurations. Secondly, potential misalignments at key connection points can be identified and quantified in terms of a probability distribution of variation. Finally, design improvements can be achieved by comparing alternate construction processes to mitigate the risk of assembly rework.

1. Introduction

A current challenge faced by many contractors whose work involves prefabrication and offsite construction is how to mitigate challenges associated with tolerance management and dimensional variability [1,2]. The authors have worked directly with several contractors in the commercial and industrial construction sectors who frequently face geometric and tolerance-related conflicts in fabrication, assembly, fit-up and erection of prefabricated assemblies. Rausch [3] provides several detailed examples of tolerance-related challenges faced on projects involving prefabrication and modularisation. These examples, and others [4,5] demonstrate the ongoing challenge in construction for addressing dimensional variability in a systematic way.

As new technologies continue to emerge and progress within the Architecture, Engineering and Construction (AEC) industry, it is becoming preferable to overcome technical challenges through use of virtual tools and si5mulation techniques. While there can be large upfront costs to acquire, train with and use virtual tools and simulation techniques, they are beneficial for analysing large datasets and simulating expected interactions and outcomes. This can be achieved at a fraction of the cost and time of traditional methods [6,7]. These cost and schedule savings are largely the result of addressing design and construction issues in a proactive manner, where the impact of resolving such technical conflicts and challenges during prefabrication is far less than encountering them during construction stages.

One proactive approach to mitigating dimensional variability issues is to coordinate design intent through building information modelling (BIM). Clash avoidance and integrated project delivery are two features of BIM that aim to proactively mitigate potential design risks, such as dimensional control issues [8]. Studies into the root causes of clashes and tolerance problems reveal that design uncertainty, lack of specificity, design complexity, and design errors can all contribute to clashes and dimensional control issues [9,10]. Park et al. [11] present a proactive approach to defect management (included dimensional

---

[*] Corresponding author at: 200 University Ave W, Waterloo, ON N2L 3G1, Canada.
E-mail address: ctrausch@uwaterloo.ca (C. Rausch).

https://doi.org/10.1016/j.autcon.2019.03.026
Received 5 November 2018; Received in revised form 21 February 2019; Accepted 30 March 2019

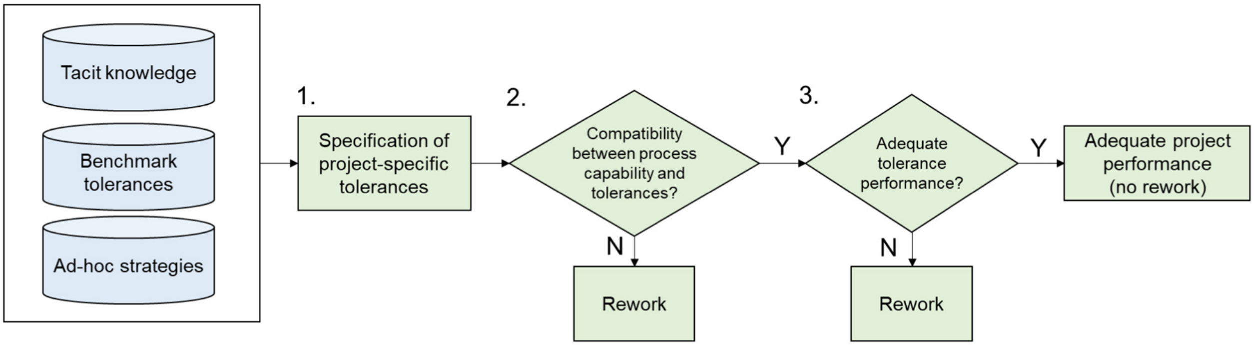

Fig. 1. Mechanism for rework associated with tolerance specification, simplified into three processes: 1. Determination of tolerances, 2. Compatibility between tolerances and process capability and 3. Adequacy of tolerance performance.

control defects) that is achieved through use of ontology-based data collection, BIM and in-field visual tools. This approach is shown to be effective for targeting defects, identifying root causes and achieving defect control with efficient communication of BIM design intent (through augmented reality and image matching). However, while proper design coordination and communication can be used to avoid gross dimensional errors (such as large misalignments of doors and windows), these approaches do not have the granularity required or ability to assess the root cause of complex dimensional variability issues. This is because that even despite its ability to be used proactively, BIM currently lacks the ability to analyse the implications of incorrectly specified tolerances [12].

The authors have previously demonstrated how analysis of as-built data (e.g., from lidar laser scanners) can be used to track dimensional error propagation in prefabricated assemblies [3]. While this approach to quantifying and diagnosing dimensional variability issues in prefabricated construction is functional, it relies on manual processing and is still reactive by nature. While ongoing research into scan-to-BIM and scan-vs-BIM processes is focused on increasing automation efforts [13–16], use of these methods is often applied reactively, or as part of a dimensional quality assurance step. They do not provide designers with proactive insight into optimal management of dimensional variability. Several studies allude to the efficacy of solving tolerance problems upstream during design rather than downstream once construction has commenced [12,17]. For this reason, there is a need to address dimensional control through use of design-based strategies and tools.

The authors are exploring how analytical tools and methods from the manufacturing industry can be applied to industrialised construction, given the parallels between these two industries [18,19]. Through this research, simulation-based tolerance analysis has been identified as a superior method for mitigating the impact of dimensional variability. Previously, studies have delved into topics surrounding tolerance management in construction including tolerance mapping, tolerance analysis, tolerance allocation, and tolerance-based constructability analysis [20–22]. While effective, these approaches require a fundamental knowledge about manufacturing tolerance theory (e.g., geometric dimensioning and tolerancing (GD&T) and complex ontologies). In the authors' experience working with contractors in the construction industry, the use of complex methodologies from manufacturing can be very challenging to incorporate into current construction workflows. This is due to the steep learning curve involved and subsequent effort to implement such workflows, which need to be tailored to unique processes and design considerations in construction. However, tolerance analysis through Monte Carlo simulation has many benefits for use in prefabricated and offsite construction. It does not require the same knowledge about complex ontologies that other methods do. Intricate geometric interactions between components in an assembly can be efficiently handled by a 3D-CAD simulation engine. Finally, optimisation of tolerance performance can be achieved by analysing the impact of alternate fabrication processes.

To the authors knowledge, no studies to date have successfully demonstrated the use of simulation-based tolerance analysis in construction for proactively managing dimensional variability. The proposed methodology presents the novel application of Monte Carlo simulation to large prefabricated construction assemblies to analyse, predict and optimise tolerance performance. The following objectives are outlined for this methodology:

- Modelling tolerance accumulation in a timely manner for prefabricated structures.
- Utilising a tolerance analysis method that can handle complex 3-D interactions between components.
- Comparing alternative construction processes to mitigate risk of assembly rework.

The required inputs of the proposed framework include the nominal assembly geometry (e.g., a 3-D CAD model), the sequence of fabrication and assembly processes, connection tolerances and critical measurement points on the assembly. The benefits of implementing the proposed methodology are summarised as: (1) rework avoidance for conflicts related to dimensional variability, (2) analysis and comparison between construction processes for optimised geometric performance, and (3) rework risk quantification and mitigation.

2. Background

2.1. Rework related to dimensional variability in construction

Due to the current fragmented practice of tolerance specification in construction [23], additional sources of dimensional variability in prefabricated and offsite construction (i.e., loads from transportation and fit-up on site) increase the risk of rework [24]. Examples of rework risks include components being too small, too large, not level, excessive geometric changes, misalignments, and assembly fit-up conflicts [25]. To explain how rework can occur, a simple mechanism is used (Fig. 1) which is synthesised from current industry practice for tolerance specification and previous research on tolerance performance. In this case, rework can be linked to incompatibility between process capability and specified tolerances, or from poor tolerance performance. The following sections delve into the core components of Fig. 1. The aim of the proposed methodology is to quantify and mitigate rework risk associated with poor tolerance performance using Monte Carlo simulation.

2.1.1. Current state of tolerance specification in construction

While dimensional tolerances for prefabricated structures should comply with values outlined in governing standards [26], a range of additional resources are often relied upon since baseline tolerances from standards and other conventional methods are not strict enough for ensuring adequate alignment between prefabricated assemblies [2,27]. Engineers and designers must rely on tacit knowledge, libraries of case-specific tolerances, and ad-hoc strategies to derive tolerances for prefabricated structures. Since reliance on these resources does not always produce adequate tolerances, assembly geometry is still corrected during construction rather than being proactively addressed during design. This is why the current state of tolerance specification is described as an inefficient and reactive process [17,25], which is further compounded by the fact that associated rework typically delays activities along the critical path of a project.

2.1.2. *Compatibility between tolerances and process capabilities*

Process capabilities define the expected variation of a given process (e.g., steel frame welding, rebar placement, concrete pouring, component alignment, etc.) which in turn can be used to determine its probability of not exceeding required tolerances. A previous study demonstrates the importance of compatibility between construction processes and tolerances for ensuring that an assembly can be fabricated, assembled and installed on site correctly [28]. This is also evident in research on Design for Construction [29], which aims to characterise construction process capabilities in order to inform design decisions. In design, compatibility between tolerances and process capability is achieved using one of two approaches: tolerance allocation or tolerance analysis. Tolerance allocation can be illustrated using an example of a steel beam that has a specified tolerance of 3 mm. The processes affecting the length of that beam (e.g., cutting, measuring, grinding, etc.) must have a net variation less than 3 mm. In this case, the tolerance can be divided, or "absorbed" between compounding processes, however the net variation of processes is what matters and must be less than the specified tolerance for the length of the beam (Fig. 2a). On the other hand, tolerance analysis occurs where process capabilities are analysed to derive a suitable overall assembly tolerance. This can be illustrated by the amount of adjustability required for the connection of a prefabricated curtain wall system. The variability of the underlying building substrate as well as positional variability of the curtain wall must be analysed in order to derive suitable tolerances (Fig. 2b). Both tolerance analysis and tolerance allocation require some knowledge about the capabilities of processes in terms of their dimensional variability. While it may be difficult to determine the dimensional variability of construction processes, Milberg and Tommelein [4] demonstrate that failure to consider process capabilities can result in conflicts during installation onsite.

2.1.3. *Interaction of tolerances on the overall assembly*

Notwithstanding the ability for processes to meet their required tolerances, another fundamental issue is ensuring that the interaction of tolerances of the overall system does not create conflicts. For instance, if a series of prefabricated modules being installed in succession to each other are subject to an overall tolerance requirement, the accumulation of tolerances in and between each module must not exceed the overall tolerance of the system (Fig. 3). In manufacturing, this example is referred to as a tolerance chain, and there are specific rules for ensuring a system has been properly "toleranced" [30]. However, the construction industry does not regularly "tolerance" assemblies since it is commonly assumed that this can be avoided through mere compliance with design and building codes, which are often not strict enough nor tailored to specific geometric requirements [12]. There is also a prevalent viewpoint in the industry that assembly and fit-up issues should be solved onsite (where costs and delays are the highest) rather than upstream in design and prefabrication stages [17]. Fortunately, the authors note that this common viewpoint is slowly shifting as the adoption of technology is increasing within construction and as the capabilities of virtual tools continue to improve.

2.2. *Virtual assembly*

Building a prototype of an assembly can be used to assess geometric compliance, "assemblability" and to understand practical design limitations. While virtual assembly simulation is widely used in manufacturing, it is only used in very specific construction applications. Case et al. [31] suggest the costs associated with virtual trial assembly of a steel structure can be 10% of the cost and 30% of the time required for physical trial assembly. This virtual prototyping process can bring significant cost and time savings for construction of complex structures [32] or for prefabrication and offsite construction where the risk of large rework costs and delays can become very significant due to economies of scale. The geometric challenges involved with construction of large assemblies can be modelled and then subsequently addressed virtually, in order to change vital details before construction.

There are two general approaches used for modelling part interaction and geometric behaviour in virtual assembly: (1) constraint-based modelling and (2) physics-based modelling [33]. In constraint-based modelling, external spatial constraints can be placed on component geometry, or constraints can be intrinsic, taking on the form of kinematic transformations of rigid bodies. In physics-based modelling, material properties and behaviour are accounted for, and is often used for real-time simulation purposes. Methods for virtual assembly and planning focus on evaluating assembly processes early in the design stage so that practical considerations and changes can be generated in a timely manner rather than bearing large unnecessary costs during construction. This is particularly applicable for the development of construction tolerances, since the overall accumulation of various

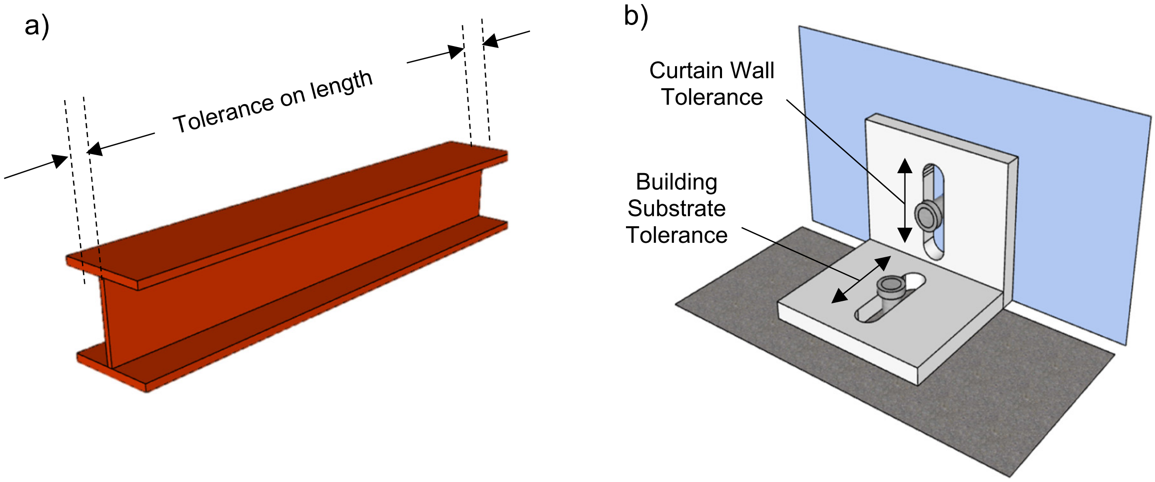

Fig. 2. Examples of tolerance design approaches: (a) tolerance allocation for the variability on the size of a steel beam and (b) tolerance analysis of a connection for a curtain wall system.

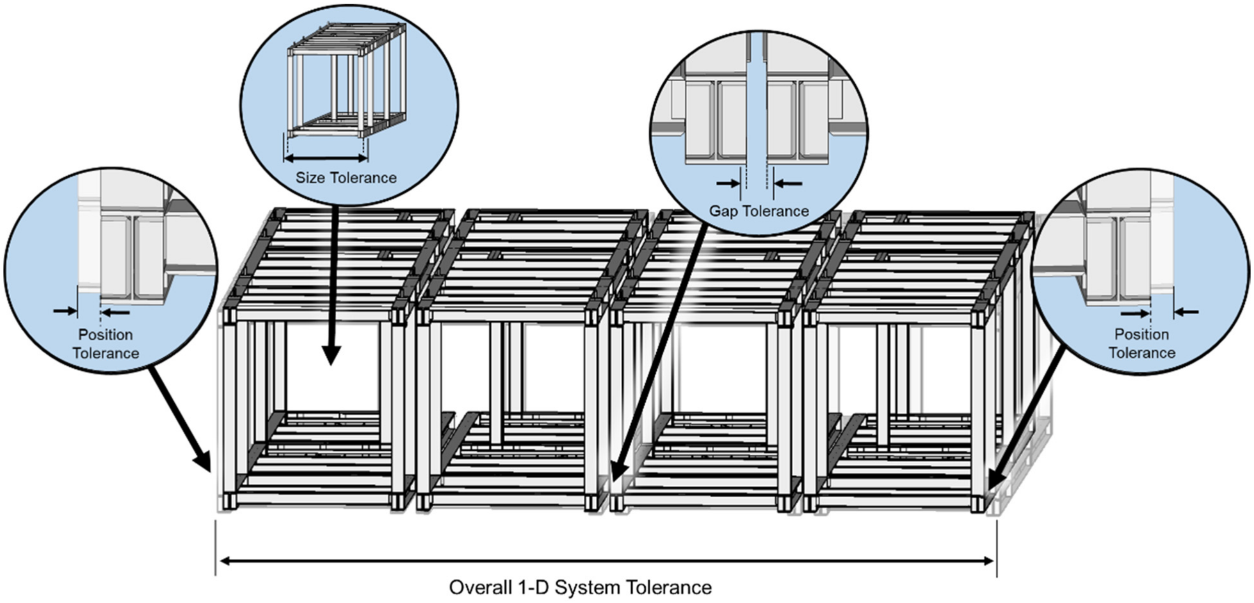

Fig. 3. Interaction of tolerances for a modular structural system (position, size, and gap tolerances) which is subjected to an overall one-dimensional system tolerance.

system tolerances can attribute to geometric conflicts.

2.3. Monte Carlo simulation for tolerance analysis

As previously evidenced in literature, tolerance analysis is a favourable method for proactive dimensional control. In manufacturing, the design of mechanical assemblies must include a dimensional analysis as part of a complete quality assurance program [34]. Tolerance analysis methods have been developed over time to address assemblies that range in complexity. Common methods include dimensional tolerance chain models (or tolerance charts), statistical tolerance models, kinematic chain analysis (or vector loop models) and Monte Carlo simulation. It is useful to categorise these methods in terms of their ability to address varying degrees of assembly complexity: methods for 1-D or 2-D analysis and methods for 3-D analysis.

Dimensional tolerance chain models aim to characterise the accumulation of tolerance in an assembly using a linear equation with worst case tolerances or using a root sum square statistical equation. The challenge with tolerance chain models is that they are usually overly conservative (worst-case tolerance chains), or overly ambitious (root sum square) in their accumulation predictions [35]. To improve accuracy, statistical tolerance methods were introduced, comprising an equation of random variables that are solved using advanced techniques (e.g., Taylor series approximation, Croft's method, Hasofer-Lind index method, higher-order integration techniques, Taguchi's method, etc.). However, the increased accuracy requires developing and solving an assembly equation that can be very complex and not practical for many real-world applications [36]. Accordingly, due to limitations in either accuracy or analytic difficulty, tolerance chains and statistical models are often confined to only 1-D or 2-D tolerance analysis.

3-D tolerance analysis is more appropriately addressed using either a kinematic chain (or vector loop) analysis or simulation-based analysis. Kinematic chain analysis characterises an assembly as a system of vectors connected at joints with translational and rotational degrees of freedom. Since kinematic chain systems are distilled into a series of matrices that often result in a non-linear equation, using them relies heavily on experience and insight [37,38]. Accordingly, simulation-based tolerance analysis is used when other methods are too cumbersome (e.g., statistical models or kinematic chain analysis) or do not provide sufficient accuracy (e.g., tolerance chains) [39]. Statistical simulation is carried out using the Monte Carlo technique which compiles stochastic samples of probability distribution functions for individual components in the assembly. Despite the criticism of its potentially high computational demand, Monte Carlo simulation has been shown to be one of the simplest and most popular methods for solving complex tolerance analysis [40,41]. In addition, research has proven the ease with which alternative production processes can be evaluated using the Monte Carlo method for optimal tolerance performance [42]. While the application of Monte Carlo simulation was initially limited to only dimensional tolerances, Yan et al. [40] developed an approach where geometrical tolerances (e.g., profile, planarity, roundness, concentricity) can be incorporated into the simulation process. This is particularly useful when trying to simplify the tolerance analysis process. Similarly, Sleath [43] describes a process of reducing an overall assembly into a subset of mating components that is representative of the assembly in terms of key geometric behaviour. These recent developments have helped to simplify large complex assemblies and to reduce the computational demand required when using Monte Carlo simulation.

Documented literature for 3-D tolerance analysis in construction is very limited. Previously, the authors demonstrated how kinematic chain analysis can be used to analyse dimensional variability in construction assemblies [44]. While this method was found to be very accurate for analysis in 3-D (predicting dimensional variations within 1% of the actual observed deviations), it relied on deep insight and experience to create an appropriate assembly equation and solve it correctly. In addition, numerous assumptions are required to use it, which limit its use for complex analysis. As such, the authors concluded that simulation-based approaches should be explored to improve the ease of 3-D tolerance analysis. Only one known study has explored the Monte Carlo method for a 3-D tolerance analysis in construction [45]. However, this approach relied on a time consuming and tedious process of developing part maps, assembly diagrams and a vector loop model, which diminishes the ease with which the Monte Carlo method is capable of being used. In addition, this approach was also unable to find reliable solutions. As a result of the challenges encountered, this study concluded that future research was still required to better validate the use of Monte Carlo simulation for accurately predicting tolerance failure rates in construction.

This article addresses the gap in literature by demonstrating how to

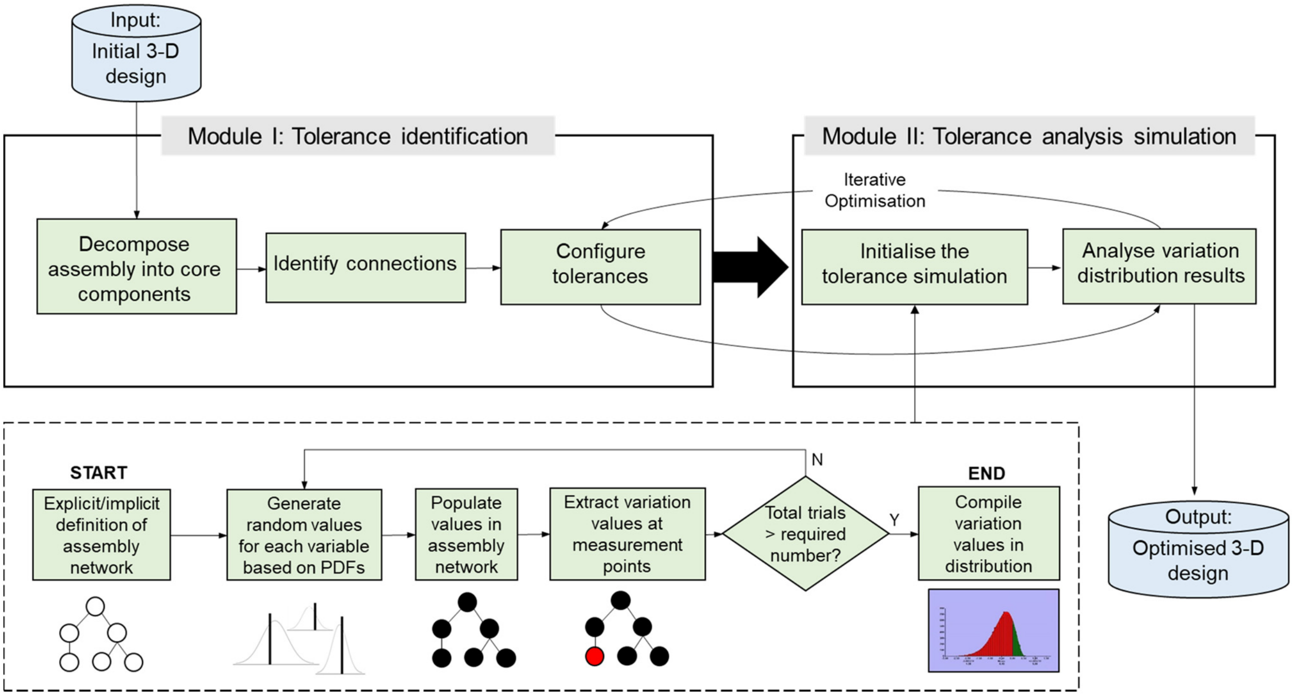

Fig. 4. Overview of the proposed framework for simulation-based tolerance analysis of construction assemblies.

apply 3-D Monte Carlo tolerance analysis in the context of construction-scale fabrication, which qualitatively differs from manufacturing environments in terms of scale, absolute values of deflections, ability to control fabrication geometrically, and impact of handling such as lifting and transporting, on the dimensional tolerance performance. As well, developments that simplify the Monte Carlo method for more effective use in construction fabrication situations, that are typically less repetitive than those for which more complex manufacturing simulations, are justified, developed and presented (i.e., simplifying the assembly into a set of components that are representative, and converting dimensional tolerances into statistical tolerances). The proposed methodology is suitable for comparing alternative production processes in order to statistically quantify and mitigate risk of rework, which is still a perennial problem in prefabricated and offsite construction.

3. Methodology

The proposed framework for tolerance analysis is shown in Fig. 4 and has two primary modules. Module 1 is the tolerance identification process that involves decomposing the overall assembly into its core components, identifying the connections between these components and configuring the corresponding tolerances at these connections. The second module is the tolerance simulation process, where results are expressed as probability distributions at critical measurement points. While the input to the framework is an initial 3-D design (including fabrication processes, tolerances and a 3-D model), the output is a design that is optimised for tolerances and risk management. The success of the optimisation process relies on the iterations involved with specifying part tolerances and fabrication processes that result in acceptable deviations at the critical measurement points.

3.1. Module I: tolerance identification

The first step is decomposing an overall assembly into its core components or subassemblies. Determining the amount of decomposition in an assembly can be a heuristic process, but ultimately depends on the desired granularity of analysis. For prefabrication and offsite construction, common connection types include bolted connections, welded connections, fittings, and anchor bolt connections. Due to the stricter tolerance requirements associated with bolted connections (as opposed to welded connections), they are examined in detail in this research. When comparing an offsite construction assembly to an assembly produced in different manufacturing industries (e.g., aerospace or automotive assemblies), it is not necessary to capture the same quantity or level of detail for part tolerances, since tolerances in construction are generally not as strict as for manufacturing [46]. While there are some exceptions to this, such as assemblies in nuclear facilities that rely on extremely tight tolerances [32], the majority of construction tolerances can be specified on the scale of several millimetres to centimetres rather than on the sub-millimetre scale. To reflect this simplification, the following types of tolerances are used in the proposed framework:

- **Size tolerances**, which express the difference between the actual length or width of a component and its nominal length or width.
- **Form tolerances**, which relate to the straightness of a linear feature (1-D profile of a component), or flatness of a surface. It should be noted that form tolerance is expressed as a linear dimension corresponding to the largest Euclidean distance between the profile of an actual line/surface feature to the nominal feature.
- **Positional tolerances**, which relate to the accuracy in location and are measured as a two-point distance from the actual position to the nominal position.

When given no prior knowledge about a specific manufacturing process, it is generally assumed that its tolerance follows the normal distribution [47]. Previous tolerance considerations in construction have also relied on the normal distribution [4]. Since most construction tolerances are specified using a numerical ± value, it is necessary to explain how numerical tolerances can be expressed as statistical tolerances which are required for statistical simulation. In accordance with principles of six-sigma methodology [48], a range of 6σ (six standard deviations) accounts for 99.73% of the entire normal distribution function. By expressing this statistical range as ± 3σ centred on the mean of the standard distribution, it is possible to associate an initial ± tolerance to ± 3σ of the normal distribution function (Fig. 5).

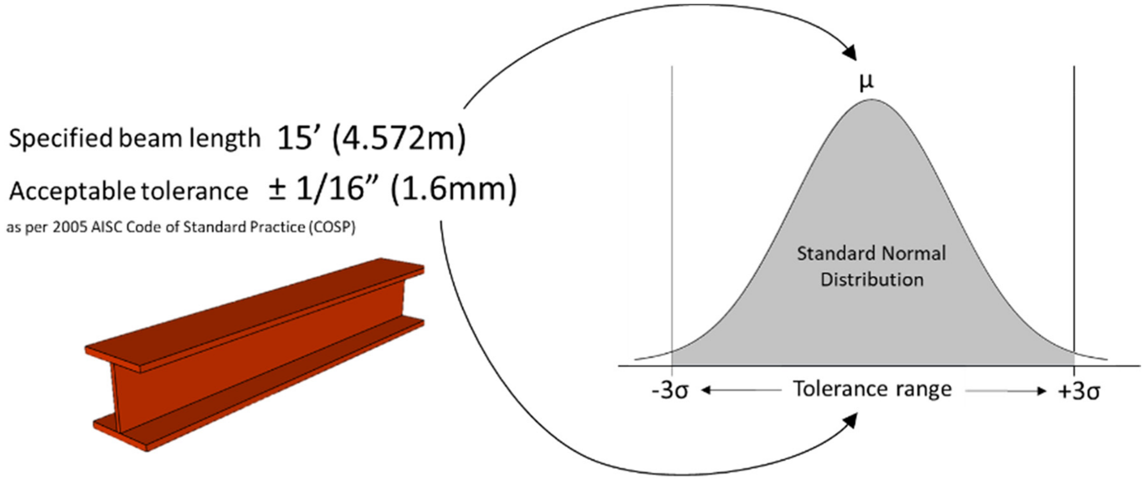

Fig. 5. Demonstrating how numerical tolerance limits can be expressed in terms of statistical tolerance limits in accordance with six sigma principles.

For tolerances following the normal distribution, two terms are used to express the tolerance configuration at a connection. Range is the spread of the distribution and offset is the mean shift of the spread. In this way, a tolerance of ± 2 mm has a range of 4 mm and an offset of 0 mm from the mean. Offsets are used to facilitate unique connection requirements. An example of this could be tolerance on standoffs used to mount balconies on a building. To ensure the balconies are installed in a vertical plane with a minimum interface gap to the building, standoff lengths can be specified with tolerance limits of 0 mm to +10 mm (range of 10 mm, mean offset of +5 mm). If a given standoff length exceeds the specified dimension, any excess length is "absorbed" by an opening at the end of the balcony connection. However, if these standoffs are shorter than the specified length, the balcony connection is such that it cannot close the gap to keep the balcony in a true vertical plane while also maintaining the minimum gap to building requirement. Therefore, a mean shift of +5 mm on the tolerance ensures installation requirements are met. As this example demonstrates, unique tolerance configurations can be facilitated through use of very basic terms of range and offset.

3.2. Module II: tolerance analysis simulation

The general procedure for Monte Carlo simulation is shown in Fig. 4 and involves the following steps: (1) define the assembly network, (2) generate random values for each variable based on their probability distribution function (PDF), (3) populate the assembly network with generated random values, (4) compute and extract variations at critical measurement points in the assembly network (5) repeat steps 2 to 4 until the number of simulations exceeds the minimum required. After each of these steps is completed, variation results can be compiled into a histogram for extraction of key sample statistics (i.e., mean and standard deviation).

The assembly network defines how each of the components is geometrically related. This process results from the decomposition of an assembly into the core subassemblies, as previously discussed, and results in a network of variables. The generation of the assembly network can be done explicitly (refer to [44] for demonstration of this using kinematics chains) or can be done implicitly using a 3D-CAD simulation engine (as was used in the functional demonstration shown in this paper). Simulating non-rigid body deformations must be done by a "net rigid body effect", through use of an equivalent dimensional variation. A key aspect in the assembly network creation is the inclusion of critical measurement points on the assembly, which are used to measure the overall compliance of the simulation process. For prefabricated and offsite construction assemblies, the following types of critical measurements can be included in an assembly network:

- **Gaps**: these can either be functional (i.e., required gaps between certain components), or can be resultant (i.e., gaps that should not be present, but are acceptable within certain limits).
- **Circle interference**: used to define acceptable alignment condition for bolted connections. This type of measurement is covered in detail in a functional demonstration and is depicted in Fig. 10.
- **Feature position**: used to define conditions of fit (i.e., the absolute or relative position of a feature of one assembly to the feature of another assembly), or are used to define appropriate inspection requirements for the overall geometry of an assembly (refer to [49] for description of maximum permitted geometric errors in the manufacture of prefabricated assemblies for high-rise buildings).

Through use of the Monte Carlo technique, the assembly network is then populated with values using a random value generator for each simulation trial. The accuracy of the simulation process is related with the number of simulation trials. It is generally very difficult to compute the minimum number of simulations required, since this depends on the complexity of the deterministic model, variance of input and required accuracy of output. However, since the Monte Carlo method is a statistical measure, the number of required simulations can be roughly estimated using the central limit theorem and the confidence bounds of the normal distribution according to [50] as,

$$N = \left(\frac{Z_{\frac{\alpha}{2}} \left(\frac{\sigma}{\mu} \right)}{\frac{Er(\mu)}{\mu}} \right)^2 \tag{1}$$

where N is the number of simulations, $Z_{\frac{\alpha}{2}}$ is the standard normal statistic, σ is the sample standard deviation, μ is the sample mean, $Er(\mu)$ is the standard error of the mean and the expression $\frac{Er(\mu)}{\mu}$ is the margin of error in percent format. The challenge when calculating the minimum number of Monte Carlo simulations is that an initial set of simulations must be run in order to obtain the sample mean and standard deviation before a final set of simulations are run. After running this second set of simulations, the minimum number of simulations should be recalculated to verify that the increased precision of the sample statistics has not changed the accuracy of the initial estimation. To avoid this potentially cumbersome and iterative process of calculating the minimum number of simulations, this research employs a different approach. Instead, a predefined number of simulations are used, and the resulting margin of error is calculated and analysed. If the resulting margin of error is deemed unacceptable, then a subsequent set of simulations can be carried out. Previous research has found that a limit of 10,000 simulations is generally sufficient for the purpose of tolerance simulation [51]. Using this value, and a predefined confidence interval, Eq. (1) can be rearranged to solve for the margin of error:

$$\left(\frac{Er(\mu)}{\mu}\right) = \frac{Z_{\frac{\alpha}{2}}\left(\frac{\sigma}{\mu}\right)}{\sqrt{N}} \tag{2}$$

The confidence interval can be obtained by solving for the significance level α, using a Z-statistic table. For the purpose of this study, a confidence interval of 95% and 10,000 simulations are used. The only drawback to this approach of calculating the margin of error is that employing a predefined number of simulations can be potentially computationally expensive. However, the simplifications employed in this methodology (e.g., reducing an overall assembly into a smaller subset of mating components), reduce the computational demand of Monte Carlo simulation as evidenced in the functional demonstration, where simulation results are obtained very quickly. It should be noted that while the approach of using a predefined set of simulations (i.e., 10,000 simulations) is adopted in this research, in cases where the simulation process is highly complex or found to produce very long run times, more advanced error estimation methods such as the Mean Square Pure Error (MSPE) evolution curve analysis approach may be required.

Lastly, the output of the simulation process is a set of distributions for each measurement variable that describes the performance of the selected tolerance and process capability. In addition to predicting the overall 3-D tolerance accumulation through Monte Carlo simulation, the proposed methodology can be used to conduct iterative optimisation by selecting different tolerances and processes in order to meet required measurement criteria. Risk of misalignment in each configuration can be measured using the statistical distributions of the tolerance simulation. By comparing different tolerances and processes, rework risk mitigation can be proactively carried out. This is covered in detail using a functional demonstrated which is presented in the following section.

4. Functional demonstration

3DCS Variation Analyst is tolerance simulation software by *Dimensional Control Systems* which was used in this paper to simulate the geometric aspects of fabrication and assembly of a modular building. Manufacturers in numerous industries including aerospace, automotive and electronics use software such as *3DCS Variation Analyst* to optimise processes, resolve manufacturing issues and control rework and scrap. The basic workflow for tolerance analysis using *3DCS Variation Analyst* was adapted from Fuentes et al. [52] and is shown in Fig. 6.

4.1. Background on modular construction project

During a modular construction project which was introduced in Rausch et al. [44], several challenges emerged during the fabrication and assembly of prefabricated structures. These challenges stemmed from dimensional variability issues that arose as the result of improper tolerance specification and geometric controls during construction. The accumulation of dimensional variability ultimately created large gaps between module tie-in plates and created challenges during fabrication and assembly. Data collected from this project is used to demonstrate how the proposed framework can detect and resolve geometric assembly conflicts. In this demonstration, the structural assembly of a module is assumed to be comprised of three types of subassemblies (base frame, columns and a roof frame), which are aggregated using bolted-connections as shown in Fig. 7. During this project, once the columns were installed to the base frame, a scan-vs-BIM analysis [13] was used to quantify compliance to an as-designed BIM model. As-built data from a FARO laser scanner (which has an accuracy of ± 2 mm for the scanning distance in this project) was registered using Autodesk ReCap to produce a 3-D point cloud which was then overlaid on a BIM model in CloudCompare (Fig. 8). This scan-vs-BIM analysis highlights several key column misalignments, ranging in magnitudes up to approximately 30 mm.

4.2. Simulation 1: detecting assembly conflicts

An initial tolerance simulation is used to detect the presence of immediate assembly issues that result from the interaction between the geometric configuration and dimensional process capabilities. For simplicity, the base frame and roof frame are considered to be rigid and free from significant manufacturing deviations (e.g., from welding distortion). As-built deviations for the base and roof frame used in this project were quantified through a scan-vs-BIM analysis, and were shown to be less than 5 mm (Fig. 9). The manufacturing deviations of the frames are small since the contractor utilised "fixturing tables" for fabrication. While this is effective in controlling the 2-D alignment of an assembly, there were no geometric controls for fixturing the 3-D alignment of the overall module structure (outside of measuring overall length and column verticality using tape measures, levels and laser meters). The ability to assemble the roof frame on the module therefore relies on several key tolerances: tolerances for placement of bolt holes in the base frame and columns, size tolerance of bolt hole diameter, size tolerance on bolt diameter, tolerance on the straightness of columns, and tolerance on the length of columns. To incorporate each of these tolerances into the simulation, several resources [46,53] and rules of

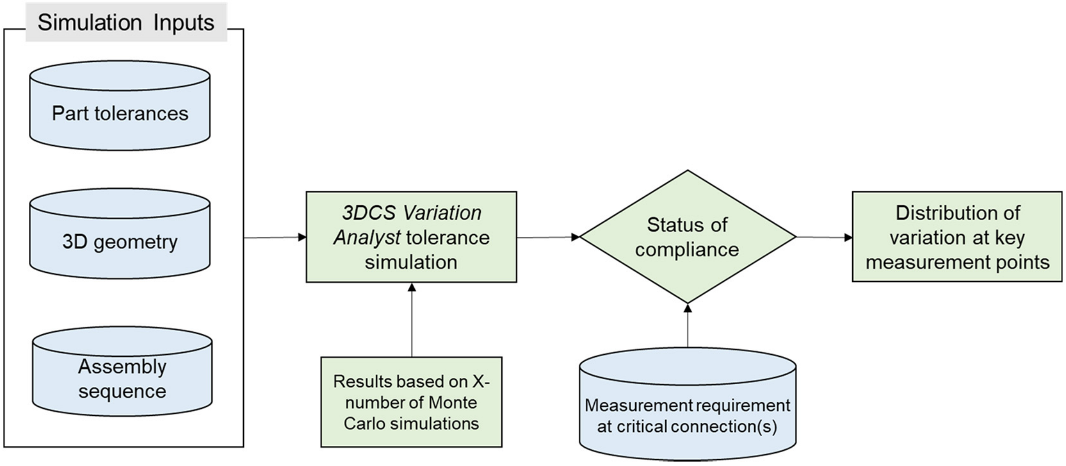

Fig. 6. Workflow for conducting tolerance analysis simulation using *3DCS Variation Analyst*.
Adapted from Fuentes et al. [52].

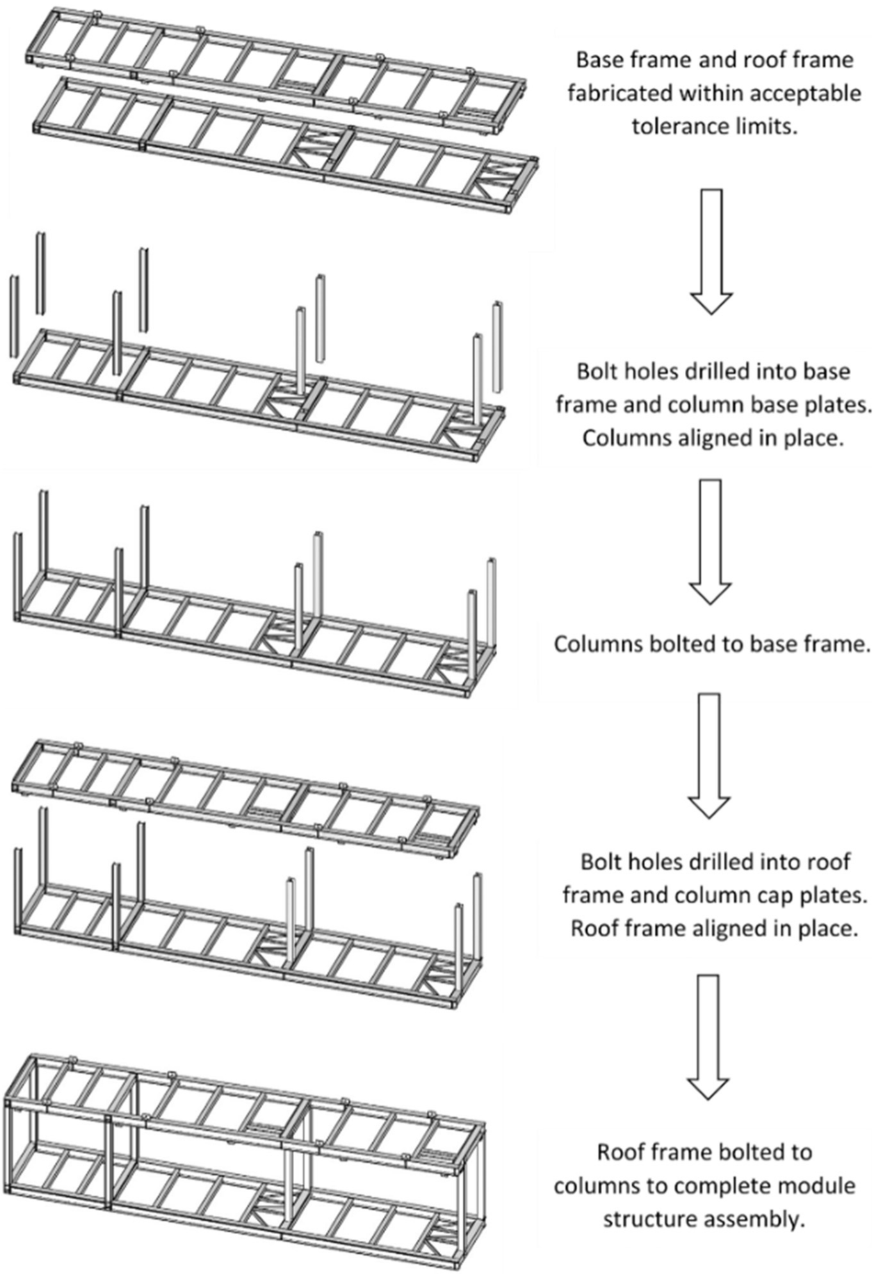

Fig. 7. Sequence of key processes for the assembly of a single module structure.

thumb were used to obtain tolerance values, distribution type and range. Each of the tolerances is assumed to be normally distributed, and have the values as indicated in Table 1. The acceptable range for each tolerance corresponds to the ± 3σ (where σ is the standard deviation) limits of the normal distribution function, which contains 99.73% of all variations.

Since the assembly sequence is characterised by key connections between subassemblies, the critical measurements for this tolerance simulation are at the bolted connections for the base frame and roof frame. In manufacturing, a specific measurement type called circle interference is used to assess the minimum clearance for pin/hole assemblies. This measurement can also be used for describing the condition of acceptable bolt-assembly alignment since it closely resembles a pin/hole assembly. In this regard, acceptable alignment occurs when there is a gap (or positive "circle interference" value) between the bolt surface and the hole surface (Fig. 10).

To account for varying degrees of rework in cases where a bolt-assembly does not align, a threshold is used where misalignments greater than 2 mm indicate a large amount of rework (realignment required), while misalignments less than 2 mm indicate a low degree of rework (where reaming or forced assembly can likely be used). While it might initially seem like a 2 mm threshold is quite small for large bolted connections in construction, it is important to note that most connections comprise multiple bolts in a specific pattern. If all bolts at a given connection experience the same misalignment vector, then a 2 mm threshold is likely to be overcome with a small amount of rework. However, to account for the condition where bolts at a given connection experience different misalignment vectors, it becomes increasingly more difficult to align them when their individual misalignments exceed 2 mm in different directions. In this simulation, the bolt diameter was ¾″ (19 mm) and the bolt hole diameter was 25/32″ (20 mm), leaving a maximum uniform gap of 0.5 mm when the bolt is perfectly centred in the bolt hole.

Based on the definition of circle interference, acceptable alignment

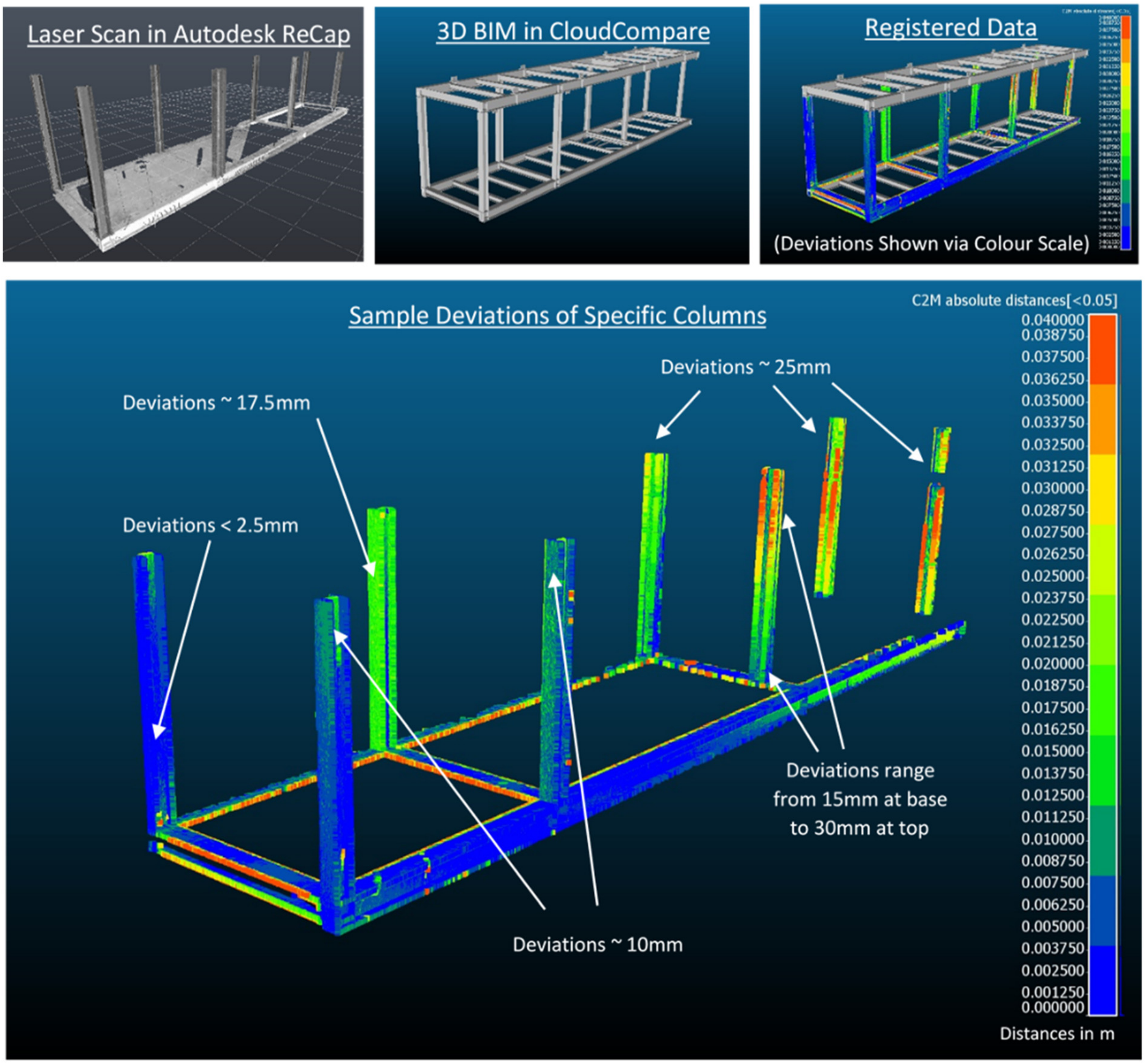

Fig. 8. Quantification of as-built deviations in module once columns were installed to the base frame. Laser scan registration was performed in Autodesk ReCap, and a scan-vs-BIM analysis was done using CloudCompare. Deviations of columns range from 2.5 mm to 30 mm.

occurs when the clearance between a bolt and bolt hole is within −2 mm and +0.5 mm. In total, 32 critical measurement points were defined in the tolerance simulation, corresponding to the top and bottom connections of the 4 outermost corner columns (there are 4 bolts at each connection). A key assumption in this analysis is that each bolt hole is drilled independently of each other (i.e., no jigs were used for hole alignment between the bottom of a column and the base frame or top of column and roof frame). Running the tolerance analysis for 10,000 Monte Carlo simulations only took 189 s and resulted in a variation distribution for each of the critical measurement points. Fig. 11 shows a sample result for the variation distribution at one bolt connection. Negative circle interference values are shown in red and represent the condition where the bolt does not fit within the bolt hole. Only values outside of the −2 mm circle interference condition represent a high degree of rework. Each variation bar chart contains the population mean, standard deviation and a best-fit probability distribution function.

4.3. *Assessing probability of rework*

In this simulation, each variation bar chart best matched a Pearson Type I distribution function, which is a generalised case of the Beta distribution (with more arbitrary shifting and scaling parameters). Normally, it is not necessary to extract the specific function pertaining to each variation distribution since tolerance simulation software such as *3DCS Variation Analyst* provides a large set of analytics pertaining to the probability of non-conformance. However, for this paper, the probability function was extracted in order to derive custom probabilities pertaining to the conditions of adequate bolt-alignment. Due to the complexity of modelling a Pearson Type I distribution, an equivalent Beta distribution was graphically fit to the data in each variation bar chart, and a cumulative distribution function was then used to derive the specific probabilities corresponding to each rework condition. The result of extracting these rework probabilities (Table 2) shows that the base connections are likely to experience only a low-degree of rework while the upper connections are likely to experience a high-degree of rework. While each of the lower connections have a mean circle interference between −0.1025 mm to 0.055 mm, the upper connections have circle interference values up to −36.68 mm, which are much greater than the limit for rework at −2 mm (note that the negative values only indicate misalignment and do not correspond to the numerical value).

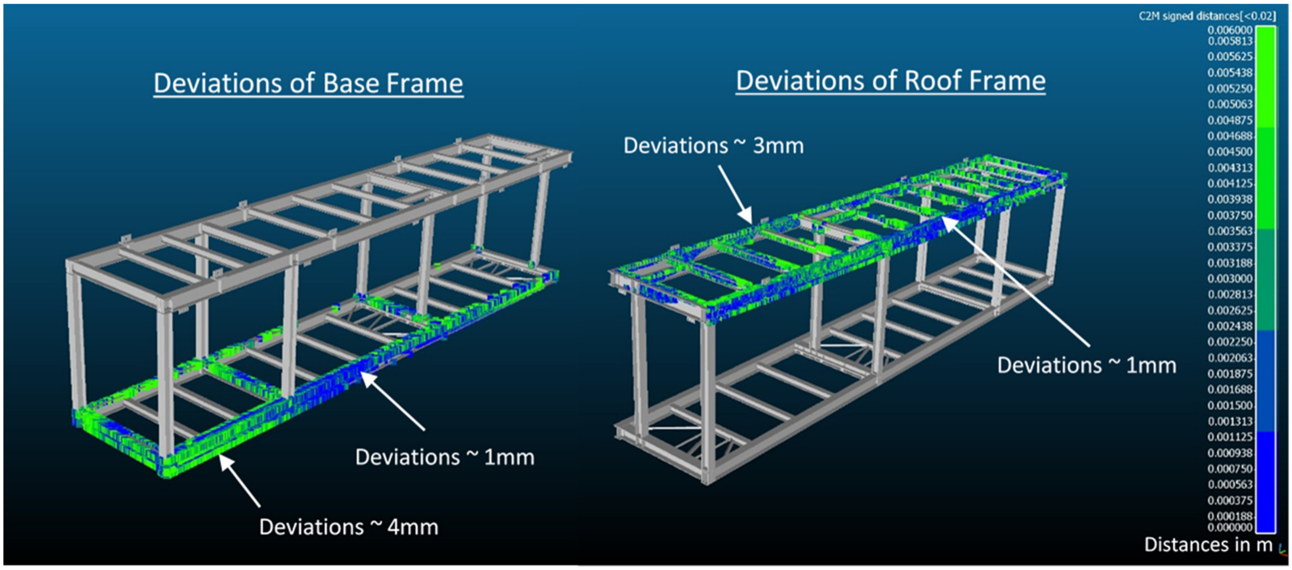

Fig. 9. Deviations of the base and roof frames for this project. Laser scans obtained for both the frames were fit to the 3D BIM model, and scan-vs-BIM deviations were obtained using CloudCompare. Deviations for fabrication of base and roof frames were less than 5 mm.

Table 1
Tolerance input values used in the simulation of the module structure assembly.

Tolerance	Tolerance type	Tolerance range (± 3σ)
Bolt hole location (drilling)	Positional tolerance	± 1/32″ (0.8 mm)
Bolt hole diameter (drilling)	Size tolerance	± 1/32″ (0.8 mm)
Bolt diameter	Size tolerance	± 5*thou* (0.12 mm)[a]
Column straightness	Form tolerance	± 5/64″ (2 mm)[b]
Column length	Size tolerance	± 5/64″ (2 mm)

[a] Since this tolerance is very small and has a near negligible effect, it was not considered in the tolerance analysis simulation.

[b] The net effect of the column straightness is a positional tolerance at the top of the column. The tolerance value corresponds to the absolute horizontal deviation at the top of the column with respect to the base.

Calculating the overall system reliability (i.e., probability that no rework is required for any connection) is complicated when accounting for the conditional probabilities between connections. For instance, there will be conditional probabilities between connections on the roof frame since a misalignment at one connection is likely to increase the chance of a misalignment at another connection (assuming the misalignment is in the opposite direction of the other connection). For simplicity by neglecting the potential conditional probabilities that may exist and only considering high-degree of rework, then the overall system reliability becomes,

$$P[rework]_{system} = 1 - P[no\ rework]_{system}$$
$$P[rework]_{system} = 1 - \prod_{i=1}^{n} P[1 - rework\ at\ connection]_i \tag{3}$$

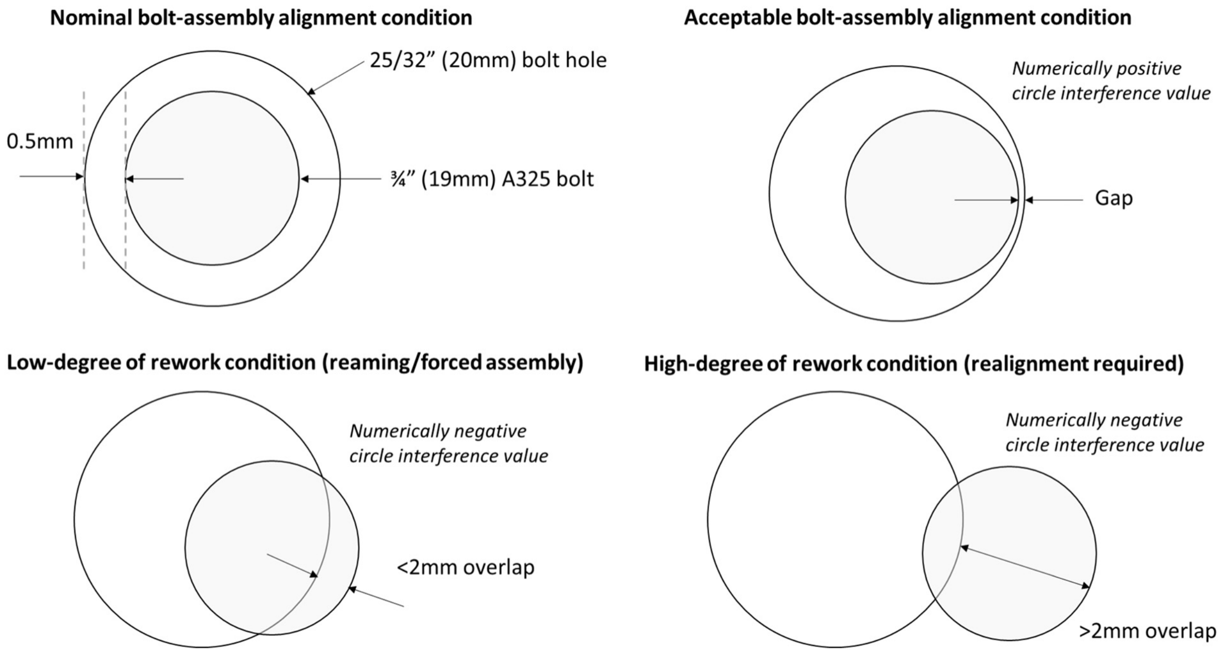

Fig. 10. Nominal bolt-assembly alignment for this functional demonstration and conditions to describe acceptable alignment and rework conditions for misalignment.

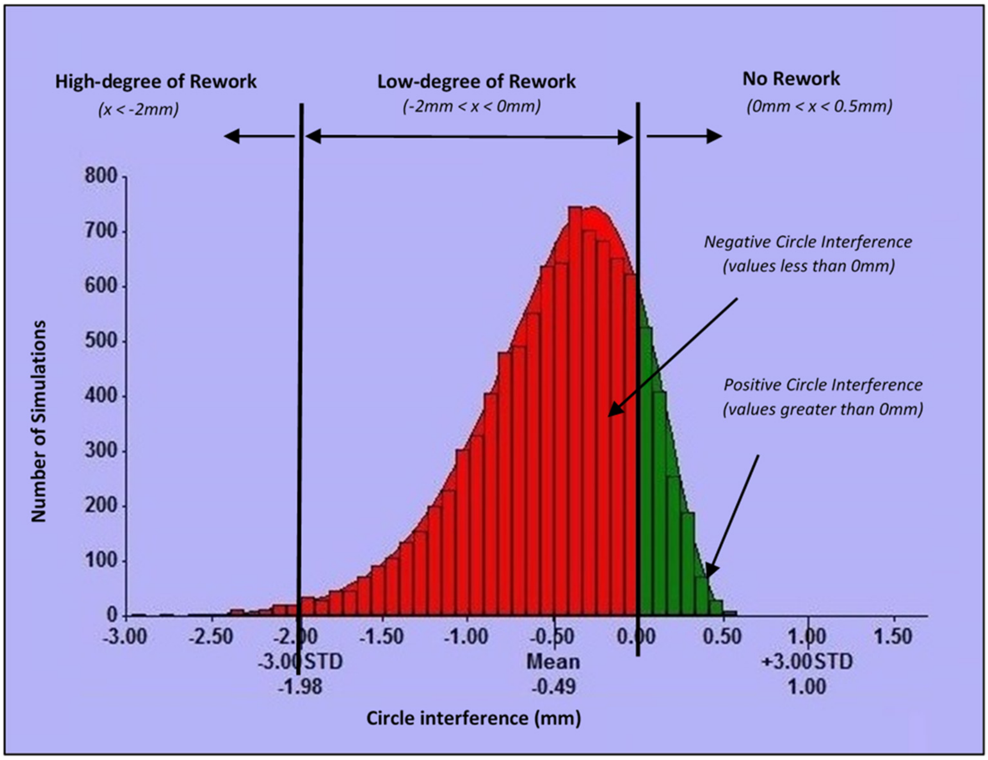

Fig. 11. Sample variation bar chart of a bolted connection. Red values indicate rework conditions, where circle interference values between −2 mm and 0 mm are a low-degree of rework and circle interference values outside of −2 mm represent a high-degree of rework. Green values indicate acceptable alignment between bolt and bolt hole, which requires no rework. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

where i ranges from 1 to 32 (for all connections), and $P[rework]_{system}$ relates to at least one event of high-degree of rework for the overall system. Calculating the probability of rework for this module with selected process capabilities results in a 99.98% probability. This is a logical result following from the specification of rework condition (circle interference outside of the 2 mm threshold) and since three connections have misalignments greater than 15 mm. To reiterate, a key assumption in this analysis is that the bolt holes are drilled independent of each other, which actually increases the likelihood of misalignment at a given connection. In practice, jigs ensure that bolt hole patterns align better between column end plates and frames. However, the purpose of this paper is to demonstrate the simulation capabilities and advantages for tolerance analysis.

The final step to this simulation is estimating the margin of error for a given confidence interval using Eq. (2). Using a confidence interval of 95%, the maximum margin of error for the governing sample statistics in Table 2 was found to equal 5.7%. Since the margin of error is linearly related to the ratio of sample standard deviation over the sample mean, the governing measurement is the one with the largest ratio (in this case Column 1 at Base). It is important to note that this method of determining the margin of error is based on an approximation of the sample statistics using the central limit theorem and confidence bounds according to the normal distribution. Since the largest margin of error for all statistics in Table 2 is 5.7%, this is the estimate of overall margin of error for this first simulation.

4.4. Simulation 2: process evaluation for optimising module assembly

A second simulation was used to reduce the high degree of rework in the first simulation by considering a new fabrication process for cutting bolt holes: water-jet cutting using a computer numerical control (CNC) machine. A cruder method of drilling bolt holes was initially selected in simulation 1. Water jet cutting is more expensive but results in a tighter tolerance with respect to the size and position of the bolt hole (0.2 mm vs 0.8 mm) due to the accuracy of a CNC machine. Following the same procedure for tolerance analysis used in the first

Table 2
Summary of results for the process of drilling bolt holes. The average values at each bolted connection are shown for the distribution mean, standard deviation, and probabilities for varying levels of rework.

Key fabrication process - Drilling bolt holes
Location tolerance = ± 1/32″ (0.8 mm), bolt hole size tolerance = ± 1/32 (0.8 mm)

Critical measurement	Average distribution[a] mean	Average distribution[a] standard deviation	Approximate probability[b] of no rework	Approximate probability[b] of low-degree of rework	Approximate probability[b] of high-degree of rework
Column 1 at Base	−0.1025	0.355	30%	70%	0%
Column 2 at Base	−0.1025	0.3525	30%	70%	0%
Column 3 at Base	−0.105	0.3525	50%	50%	0%
Column 4 at Base	0.055	0.1375	70%	30%	0%
Column 1 at Roof	−0.1725	0.375	20%	80%	0%
Column 2 at Roof	−15.395	12.1875	0%	10%	90%
Column 3 at Roof	−33.18	25.985	0%	5%	95%
Column 4 at Roof	−36.6775	28.7125	0%	5%	95%

[a] For each connection, the average distribution results for all 4 bolts are shown.

[b] Probabilities are approximated using graphical comparison to a beta distribution function.

Table 3

Summary of results for the process of water jet cutting bolt holes. The average values at each bolted connection are shown for the distribution mean, standard deviation, and probabilities for varying levels of rework.

Key fabrication process – Water jet cutting bolt holes
Location tolerance = ± 8*thou* (0.2 mm), bolt hole size tolerance = ± 8*thou* (0.2 mm)

Critical measurement	Average distribution[a] mean	Average distribution[a] standard deviation	Approximate probability[b] of no rework	Approximate probability[b] of low-degree of rework	Approximate probability[b] of high-degree of rework
Column 1 at Base	0.1575	0.09	100%	0%	0%
Column 2 at Base	0.16	0.09	100%	0%	0%
Column 3 at Base	0.1575	0.09	100%	0%	0%
Column 4 at Base	0.16	0.09	100%	0%	0%
Column 1 at Roof	0.14	0.11	90%	10%	0%
Column 2 at Roof	0.14	0.11	90%	10%	0%
Column 3 at Roof	0.1375	0.11	90%	10%	0%
Column 4 at Roof	0.14	0.11	90%	10%	0%

[a] For each connection, the average distribution results for all 4 bolts are shown.

[b] Probabilities are approximated using graphical comparison to a beta distribution function.

simulation, the result of "tightening" the positional and size tolerances for bolt holes has a profound effect on the rework associated with adequate assembly alignment. This time, running the tolerance analysis for 10,000 simulations only took 55 s. As seen in Table 3, there is no longer any probability of a high-degree of rework event for any connections in the assembly. The overall probability of a low-degree of rework event for the assembly using Eq. (2) results in a 34.4% probability, meaning that the overall likelihood of no rework for this assembly is 65.6%. Again, it is important to note that a low-degree of rework event is considered to be a minor amount of reaming and forced assembly, both of which are still viewed by contractors to be normal assembly requirements. Similar to simulation 1, the margin of error for a given confidence interval can be estimated. Using the same 95% confidence interval, the maximum margin of error for the governing sample statistics (Column 3 at Roof) in Table 3 is equal to 1.3%. As such, the estimate of overall margin of error in this simulation is 1.3% with a confidence interval of 95%.

5. Comparing Monte Carlo simulation to other tolerance analysis methods

To base the results obtained in this demonstration to other tolerance analysis methods, the following methods are examined and contrasted: a worst-case tolerance chain, a root sum square tolerance chain, and kinematic chain analysis.

As identified in literature, dimensional tolerance chain methods (worst-case and root sum square) are generally limited in their use to just 1-D or 2-D tolerance analysis. To reflect this, a simplified 1-D analysis is carried out for a worst-case tolerance chain and a root sum square tolerance chain. A cross-sectional diagram of columns 1 and 2 in the assembly depicts the accumulation of variations in the form of a 1-D tolerance chain analysis in the x-direction (Fig. 12). In this figure, column straightness, bolt hole size tolerance, nominal joint slippage (i.e., the gap between the bolt and bolt hole), and bolt hole position all contribute to the alignment between the column shown on the right and the roof frame assembly. Worst-case tolerance accumulation and root sum square tolerance accumulation are calculated according to the following equations

$$T_{WC} = \sum_{i=1}^{n} T_i \tag{4}$$

$$T_{RSS} = \sqrt{\sum_{i=1}^{n} T_i^2} \tag{5}$$

where T_{WC} is the worst-case tolerance accumulation, T_{RSS} is the root sum square tolerance accumulation, and T_i are individual tolerances for n number of chain elements. Using the chart in Fig. 12, tolerance accumulation along the 1-D chain in the x-direction results in the following calculations for tolerance accumulation:

$$T_{WC} = 2A + 8B + 3C + 8D = 19.8\ mm$$

$$T_{RSS} = \sqrt{2A^2 + 8B^2 + 3C^2 + 8D^2} = 4.6\ mm$$

where A is the column straightness, B is bolt hole size tolerance, C is nominal joint slippage, and D is bolt hole position tolerance. The tolerance values used in this calculation correspond to bolt hole drilling as the key fabrication process (as per the tolerances outlined in Simulation 1). An equivalent deviation using the Monte Carlo method between column 2 and the roof frame was found to be 15.4 mm while the as-built deviation at the top of column 2 was found to be roughly 11 mm. From these results, we can see that the worst-case tolerance chain method produces a more conservative (i.e., larger) accumulation than the Monte Carlo method, while the root sum square has an overly ambitious tolerance accumulation value. While the Monte Carlo method also produces a conservative result compared to the equivalent as-built deviation, it is important to note that in terms of tolerance analysis, conservative results are superior to ambitious results for identifying potential misalignments. The comparison of Monte Carlo with worst-case and root sum square tolerance chains match the conclusions observed in literature. In addition, dimensional tolerance chain methods do not account for rotational errors that result from deviations in higher-order degrees of freedom (i.e., rotations in the Z direction). Even slight rotational errors at connections at one end of a module can lead to significant misalignments at the far end, which are be accounted for in the Monte Carlo simulation that in essence estimates a joint confidence limit (re [54].). For these reasons, the Monte Carlo method is superior to tolerance chains for 3-D tolerance analysis on large complex structural assemblies.

In a previous study [44], we demonstrate the use of kinematic chain analysis on a prefabricated assembly that came from the same project that was explored in this article. While this assembly was not the exact same one analysed in this study, the structural configuration is very similar. In our previous study we found that kinematic chain analysis could predict 3-D tolerance accumulation between a reference datum point on the bottom of the assembly to tie-in plates on the top of the assembly, which function as "end-effectors" in a kinematic chain. Since kinematic chain analysis relies on rotational and translational degrees of freedom, accounting for more complex interactions, such as gaps at joints, is challenging. To overcome this, several assumptions and simplifications were required. For instance, the kinematic behaviour of the roof frame was confined to rotation about the vertical axis alone. Another key assumption was that the entire base frame with columns was assumed to be one subassembly with negligible tolerances. The propagation of tolerance was thus attributed to two mechanisms: (1)

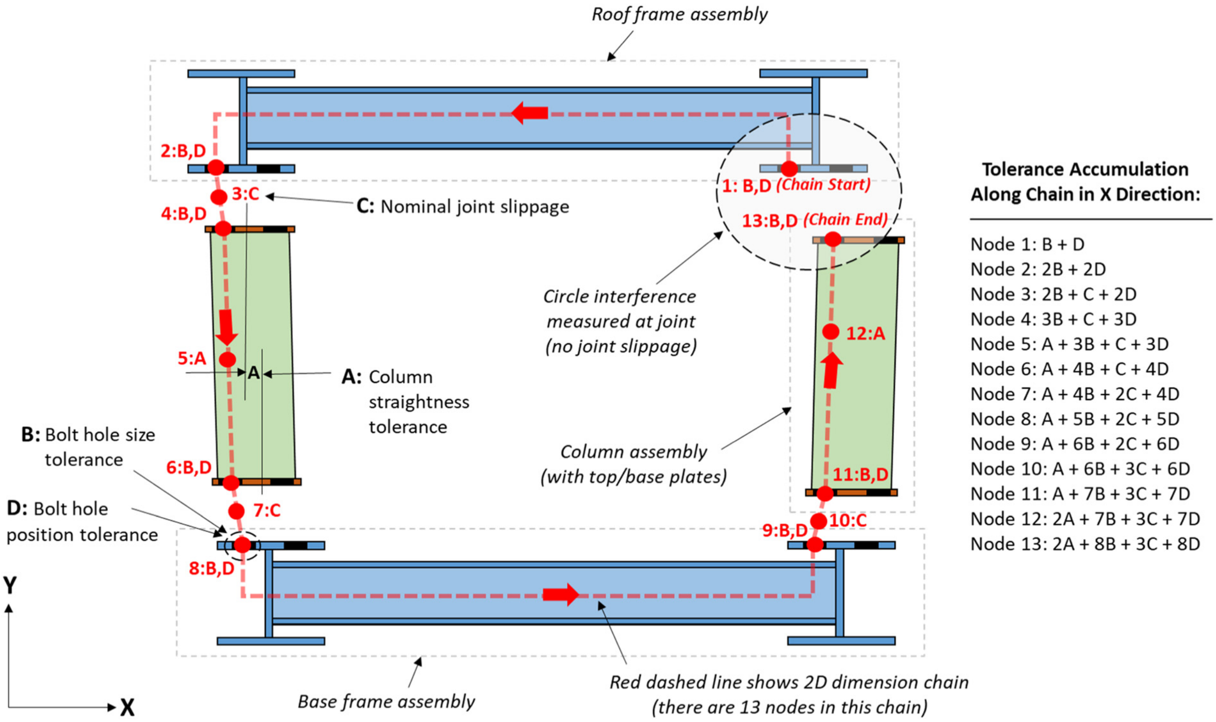

Fig. 12. Simplified accumulation of variability and tolerances using a 1-D dimension chain in the x-direction illustrated in elevation view.

deviations of each tie-in plate on the roof frame, and (2) deviation of the roof frame on the base frame and column subassembly. This means that in order to apply the kinematic chain method for the same 3-D analysis simulated using the Monte Carlo method in this study, a highly-variate kinematic chain equation would be required. Furthermore, the assumption that base frame and columns function can be taken as one subassembly is not valid for two reasons. First, the as-built deviations of the columns with respect to the base frame were shown to be non-negligible. Second, discontinuities will exist when comparing the kinematic chains between each column to the connection points on the roof frame. It is therefore too complex to use kinematic chain analysis at the same granularity of 3-D tolerance analysis that the Monte Carlo method is capable of. Similarly, the consideration of other tolerance analysis methods (e.g., tolerance mapping and statistical models) is also too cumbersome and complex to accurately predict 3-D tolerance accumulation in the way that Monte Carlo simulation can.

In summary, compared to other tolerance analysis methods, Monte Carlo simulation has many desirable attributes in the context of prefabrication and offsite construction. First, it is capable of handling complex three-dimensional relationships between components that other methods such as tolerance chain methods, kinematic chain analysis and tolerance mapping cannot. While other tolerance analysis methods may rely on the user having a fundamental understanding of graph theory and or comprehensive manufacturing tolerance nomenclature (e.g., GD&T), tolerance simulation can be carried out using simple tolerance configurations. Finally, it is shown to be a very effective design tool for comparing various fabrication processes based on tolerance performance and rework risk management.

6. Conclusions

Ongoing advancements in the construction industry are providing better ways to reduce risk, improve productivity and optimise project performance. Improvements to design are being realised through better project coordination and digitised data management which is made possible through BIM advancements. Offsite construction and modularisation are improving the delivery of buildings and assets, which is enhanced by novel production techniques such as robotic fabrication, advanced fixturing systems and additive manufacturing. While the efficacy of these emerging industry trends is evident when things go well on projects, there is still a clear need within the industry for proactive measures to characterise and manage project variability. This is especially relevant in the context of dimensional variability management of large prefabricated assemblies, which often experience rework during production or onsite to bring assemblies into proper alignment. While previous research has studied different methods to deal with this challenge, there are minimal studies that address dimensional variability proactively during design, let alone for complex 3-D analysis.

This research is the first of its kind to demonstrate that tolerance behaviour and process capability can be simulated and optimised for prefabricated and offsite construction assemblies. Conventional tolerance chain methods are either too conservative or too ambitious in their prediction of tolerance accumulation. Other methods such as kinematic chain analysis, statistical methods or tolerance mapping methods are too challenging to use given the complexity in accounting for systematic behaviour in 3-D construction assemblies. Tolerance analysis through Monte Carlo simulation is proactive, applicable for complex 3-D assemblies and can reduce rework associated with correcting geometry during fabrication and onsite assembly. Traditionally, predicting misalignments at joints was elusive due to the complex 3-D interaction between components and the inability to model tolerance accumulation in such systems. However, utilising process capability data in the form of statistical tolerance distributions, simulations can be used to model, predict and correct misalignments that may occur at critical joint locations.

A functional demonstration of Monte Carlo simulation for tolerance analysis detected misalignments up to 37 mm in a steel framed modular construction assembly with an estimated margin of error of 5.7%. The magnitude of this deviation matched the results generated from a scan-vs-BIM analysis, where as-built deviations were found to range up to 30 mm. While this degree of misalignment may not initially seem large enough to warrant a large amount of rework, very strict tolerance requirements are placed on modular construction assemblies [55]. Not only are these strict tolerances required to properly assemble modules offsite (during initial manufacturing), but the overall geometry of modules must also comply with strict alignment demands onsite. Tolerance simulation was also used to optimise the initial design through selection of an alternate production process. By tightening the tolerance associated with bolt hole creation, misalignment rework at connections is significantly reduced. While the initial design was found to have a certain probability (i.e., 100%) of a large rework event, the optimised design with tighter tolerances had only a 34% chance of a small rework event (at an estimated margin of error of only 1.3%). In this way, Monte Carlo simulation for tolerance analysis is shown to be a powerful tool for process optimisation.

As the uptake of virtual and computational tools continues in the AEC industry, it is becoming far more favourable to solve complex geometric problems in a proactive manner through use of simulation tools. Due to the complexities of dimensional variation and tolerances in large prefabricated construction assemblies, adjustable connections are sometimes used as a strategy for ensuring proper assembly requirements [56]. However, the use of proactive virtual tools such as tolerance simulation demonstrates that other strategies can be utilised to mitigate the risk of assembly rework. In contrast to adjustable connections that may require a considerable amount of engineering and design effort, evaluation of alternative fabrication processes offers increased viability in many situations. Ultimately, Monte Carlo simulation can be used as a platform for comparing these types of alternative strategies due to its versatility and ease of tolerance configuration. While this paper focused on the offsite assembly of a prefabricated structure, the proposed methodology can also be used to predict and manage dimensional variability for onsite assembly processes as well.

6.1. Limitations and future work

A limitation to this research is that all non-rigid deformation must be “converted” into a net rigid-body effect. While the impact of this assumption was not explored in depth in this article, other tolerance analysis methods also rely on this same assumption [36,57,58]. In the case of the functional demonstration used in this article, the non-rigid effect of welding distortion on the base and roof frames was quantified using an as-built analysis through laser scanning. Since there was only a minimal amount of welding distortion on the features affecting the alignment of the columns (i.e., vertical deviations on the beam flanges), these non-rigid deformations were ignored. However, if this distortion turned out to be significant, then an equivalent tolerance would have to be included in the tolerance simulation to account for the effect that it has on the accumulation of dimensional variability.

Since Monte Carlo simulation for tolerance analysis is intended to proactively manage dimensional variability, it is not practical to manually quantify statistical process capabilities before running simulations. Furthermore, use of Monte Carlo simulation should avoid tedious setup and computationally intensive processing. As such, future work should address three key areas of development: (1) incorporation of multi-physics analysis to predict material behaviour (2) development of a database to better characterise the statistical distributions of various processes and (3) techniques to improve the simulation setup and execution process.

To overcome the non-rigid body deformation assumption, future work should investigate how to incorporate multi-physics capabilities such as finite element analysis (FEA) into the simulation process. By modelling realistic material behaviour, elastic deformation could be included in the simulation process as a means of gaging the flexibility of assemblies for alignment purposes. These analyses also need to account for the structural performance of prefabricated systems since there is an inherent trade-off between geometric flexibility in a system and structural integrity. This is especially important for the design of multi-storey modular buildings.

As more studies continue to quantify process capability data in construction, it will be beneficial to characterise the statistical distributions of this data to improve tolerance analysis results. This will be useful for addressing whether certain processes need to be considered in an analysis (i.e., whether the welding distortion in steel frames is significant), but also for improving the selection of statistical distributions. While this article relied on using the Normal Distribution for tolerance inputs, improvements can be made through statistical characterisation of process capabilities. Furthermore, future work should investigate the impact of truncated distributions. Since use of non-truncated Normal Distributions (as in this research) can have an impact on the goodness of simulation results, this should be studied in more detail to determine its sensitivity.

Finally, future work should address how to improve the simulation process as a whole. Derivation of the assembly network and feeding inputs into the simulation engine can be tedious for more complex assemblies than the one examined in this research. Automated workflows offer many benefits for improving the setup of tolerance simulation. For instance, computational algorithms can be used to automatically extract the assembly network from a given BIM model of a construction assembly. As such, future work should explore how the initialisation of tolerance simulation can be improved through use of automated workflows.

Acknowledgements

The authors of this research would like to acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC) and the in-kind support of Dimensional Control Systems (DCS).

References

[1] M. Kim, Q. Wang, H. Li, Non-contact sensing based geometric quality assessment of buildings and civil structures: a review, Autom. Constr. 100 (2019) 163–179 https://doi.org/10.1016/j.autcon.2019.01.002.
[2] V.S. Kalasapudi, P. Tang, C. Zhang, J. Diosdado, R. Ganapathy, Adaptive 3D imaging and tolerance analysis of prefabricated components for accelerated construction, Procedia Engineering 118 (2015) 1060–1067, https://doi.org/10.1016/j.proeng.2015.08.549.
[3] C. Rausch, Framework for the Strategic Management of Dimensional Variability of Structures in Modular Construction, MASc http://hdl.handle.net/10012/10789, (2016) (Jan 1, 2019).
[4] C.T. Milberg, I.D. Tommelein, Tolerance and constructability of soldier piles in slurry walls, J. Perform. Constr. Facil. 24 (2009) 120–127, https://doi.org/10.1061/(ASCE)CF.1943-5509.0000079.
[5] F.J. Valdes, Manufacturing Compliance Analysis for Architectural Design: A Knowledge-aided Feature-based Modeling Framework, PhD http://hdl.handle.net/1853/54973, (2016) (Feb 9, 2019).
[6] F. Leite, Y. Cho, A.H. Behzadan, S. Lee, S. Choe, Y. Fang, R. Akhavian, S. Hwang, Visualization, information modeling, and simulation: grand challenges in the construction industry, J. Comput. Civ. Eng. (2016), https://doi.org/10.1061/(ASCE)CP.1943-5487.0000604.
[7] S. AbouRizk, Role of simulation in construction engineering and management, J. Constr. Eng. Manag. 136 (2010) 1140–1153, https://doi.org/10.1061/(ASCE)CO.1943-7862.0000220.
[8] D. Bryde, M. Broquetas, J.M. Volm, The project benefits of building information modelling (BIM), Int. J. Proj. Manag. 31 (2013) 971–980, https://doi.org/10.1016/j.ijproman.2012.12.001.
[9] I.D. Tommelein, S. Gholami, Root causes of clashes in building information models, Proceedings of the 20th Annual Conference of the International Group for Lean Construction, (2012) http://irep.ntu.ac.uk/id/eprint/22969.
[10] A.O. Akponeware, Z.A. Adamu, Clash detection or clash avoidance? An investigation into coordination problems in 3D BIM, Buildings 7 (2017) 75, https://doi.org/10.3390/buildings7030075.
[11] C. Park, D. Lee, O. Kwon, X. Wang, A framework for proactive construction defect management using BIM, augmented reality and ontology-based data collection template, Autom. Constr. 33 (2013) 61–71, https://doi.org/10.1016/j.autcon.2012.09.010.
[12] N. Puri, Y. Turkan, Toward automated dimensional quality control of precast concrete elements using design BIM, WIT Transactions on The Built Environment 169 (2017) 203–210, https://doi.org/10.2495/BIM170191.
[13] F. Bosché, M. Ahmed, Y. Turkan, C.T. Haas, R. Haas, The value of integrating Scan-to-BIM and Scan-vs-BIM techniques for construction monitoring using laser scanning and BIM: the case of cylindrical MEP components, Autom. Constr. 49 (2015) 201–213, https://doi.org/10.1016/j.autcon.2014.05.014.
[14] P. Tang, D. Huber, B. Akinci, R. Lipman, A. Lytle, Automatic reconstruction of as-built building information models from laser-scanned point clouds: a review of

related techniques, Autom. Constr. 19 (2010) 829–843, https://doi.org/10.1016/j.autcon.2010.06.007.

[15] V.S. Kalasapudi, Y. Turkan, P. Tang, Toward automated spatial change analysis of MEP components using 3D point clouds and as-designed BIM models, 2 2014 2nd International Conference on 3D Vision, 2014, pp. 145–152, , https://doi.org/10.1109/3DV.2014.105.

[16] M. Kim, Q. Wang, J. Park, J.C. Cheng, H. Sohn, C. Chang, Automated dimensional quality assurance of full-scale precast concrete elements using laser scanning and BIM, Autom. Constr. 72 (2016) 102–114, https://doi.org/10.1016/j.autcon.2016.08.035.

[17] S. Talebi, L. Koskela, M. Shelbourn, P. Tzortzopoulos, Critical review of tolerance management in construction, 4th Annual Conference of the International Group for Lean Construction, 2016 http://eprints.hud.ac.uk/id/eprint/29142, (2007).

[18] D.M. Gann, Construction as a manufacturing process? Similarities and differences between industrialized housing and car production in Japan, Construction Management & Economics 14 (1996) 437–450, https://doi.org/10.1080/014461996373304.

[19] N. Boyd, M.M. Khalfan, T. Maqsood, Off-site construction of apartment buildings, J. Archit. Eng. 19 (2012) 51–57, https://doi.org/10.1061/(ASCE)AE.1943-5568.0000091.

[20] C. Milberg, I.D. Tommelein, Application of tolerance mapping in AEC systems, Construction Research Congress, 2005, pp. 1–10, , https://doi.org/10.1061/40754(183)113.

[21] T. Acharjee, Investigating Accumulation of Tolerances and Its Impact on Reliability of Job Site Installation (MASc), https://etd.ohiolink.edu/pg_10?0::NO:10:P10_ACCESSION_NUM:ucin1172857125, (2007) (Feb 2, 2019).

[22] C. Milberg, I.D. Tommelein, Application of tolerance analysis and allocation in work structuring: partition wall case, Proceedings of the 11th Annual Conference of the International Group for Lean Construction, 2003 https://iglc.net/papers/Details/260/pdf.

[23] C. Milberg, I.D. Tommelein, T. Alves, Improving design fitness through tolerance analysis and tolerance allocation, 3rd International Conference on Concurrent Engineering in Construction, University of California, Berkeley, 2002, http://faculty.ce.berkeley.edu/tommelein/concur02/CEC02-Proceedings/16%20DS%20Milberg&Tommelein&Alves.pdf.

[24] R. Jin, S. Gao, A. Cheshmehzangi, E. Aboagye-Nimo, A holistic review of off-site construction literature published between 2008 and 2018, J. Clean. Prod. (2018), https://doi.org/10.1016/j.jclepro.2018.08.195.

[25] Y. Shahtaheri, C. Rausch, J. West, C. Haas, M. Nahangi, Managing risk in modular construction using dimensional and geometric tolerance strategies, Autom. Constr. 83 (2017) 303–315, https://doi.org/10.1016/j.autcon.2017.03.011.

[26] AISC, Steel Construction Manual, 14th ed., American Institute of Steel Construction, 2010 (ISBN 978-1564240606).

[27] C. Rausch, M. Nahangi, M. Perreault, C.T. Haas, J. West, Optimum assembly planning for modular construction components, J. Comput. Civ. Eng. (2016), https://doi.org/10.1061/(ASCE)CP.1943-5487.0000605.

[28] C. Milberg, I. Tommelein, Role of tolerances and process capability data in product and process design integration, Construction Research Congress (2003) 1–8, https://doi.org/10.1061/40671(2003)93.

[29] R. Gerth, A. Boqvist, M. Bjelkemyr, B. Lindberg, Design for construction: utilizing production experiences in development, Constr. Manag. Econ. 31 (2013) 135–150, https://doi.org/10.1080/01446193.2012.756142.

[30] B.M. Colosimo, N. Senin, Geometric Tolerances: Impact on Product Design, Quality Inspection and Statistical Process Monitoring, Springer Science & Business Media, 2010 (ISBN 9781849963107).

[31] F. Case, A. Beinat, F. Crosilla, I. Alba, Virtual trial assembly of a complex steel structure by Generalized Procrustes Analysis techniques, Autom. Constr. 37 (2014) 155–165, https://doi.org/10.1016/j.autcon.2013.10.013.

[32] F.J. Fuentes, V. Trouvé, E. Blessing, J. Cordier, J. Reich, Status of ITER dimensional tolerance studies, Fusion Engineering and Design 88 (2013) 597–601, https://doi.org/10.1016/j.fusengdes.2013.01.061.

[33] M.C. Leu, H.A. ElMaraghy, A.Y. Nee, S.K. Ong, M. Lanzetta, M. Putz, W. Zhu, A. Bernard, CAD model based virtual assembly simulation, planning and training, CIRP Annals-Manufacturing Technology 62 (2013) 799–822, https://doi.org/10.1016/j.cirp.2013.05.005.

[34] H. Navas, Monte Carlo model applied to tolerance analysis of mechanical assembly sets, American Journal of Mechanical Engineering and Automation 2 (2015) 55–58 http://www.openscienceonline.com/author/download?paperId=2653&stateId=8000&fileType=3 (Feb 9, 2019).

[35] S.D. Nigam, J.U. Turner, Review of statistical approaches to tolerance analysis, Comput. Aided Des. 27 (1995) 6–15, https://doi.org/10.1016/0010-4485(95)90748-5.

[36] Y. Hong, T. Chang, A comprehensive review of tolerancing research, Int. J. Prod. Res. 40 (2002) 2425–2459, https://doi.org/10.1080/00207540210128242.

[37] J. Gao, K.W. Chase, S.P. Magleby, Generalized 3-D tolerance analysis of mechanical assemblies with small kinematic adjustments, IIE Trans. 30 (1998) 367–377, https://doi.org/10.1023/A:1007451225222.

[38] Z. Shen, G. Ameta, J.J. Shah, J.K. Davidson, A comparative study of tolerance analysis methods, ASME 2004 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, 2004, pp. 471–482, , https://doi.org/10.1115/1.1979509.

[39] C. Lin, W. Huang, M. Jeng, J. Doong, Study of an assembly tolerance allocation model based on Monte Carlo simulation, J. Mater. Process. Technol. 70 (1997) 9–16, https://doi.org/10.1016/S0924-0136(97)00034-4.

[40] H. Yan, X. Wu, J. Yang, Application of Monte Carlo method in tolerance analysis, Procedia CIRP 27 (2015) 281–285, https://doi.org/10.1016/j.procir.2015.04.079.

[41] S. Ramnath, P. Haghighi, A. Chitale, J.K. Davidson, J.J. Shah, Comparative study of tolerance analysis methods applied to a complex assembly, Procedia CIRP 75 (2018) 208–213, https://doi.org/10.1016/j.procir.2018.04.073.

[42] S.H. Huang, Q. Liu, R. Musa, Tolerance-based process plan evaluation using Monte Carlo simulation, Int. J. Prod. Res. 42 (2004) 4871–4891, https://doi.org/10.1080/0020754042000264608.

[43] L.C. Sleath, The Dimensional Variation Analysis of Complex Mechanical Systems, https://dspace.lboro.ac.uk/2134/13996, (2014) (Eng.D., Feb 9, 2019).

[44] C. Rausch, M. Nahangi, C. Haas, J. West, Kinematics chain based dimensional variation analysis of construction assemblies using building information models and 3D point clouds, Autom. Constr. 75 (2017) 33–44, https://doi.org/10.1016/j.autcon.2016.12.001.

[45] C.T. Milberg, Application of Tolerance Management to Civil Systems, Application of Tolerance Management to Civil Systems, PhD https://pdfs.semanticscholar.org/7319/2174e99b67f20dd3175b4fefef26f5560e3b.pdf, (2006) (Feb 7, 2019).

[46] D.K. Ballast, Handbook of Construction Tolerances, John Wiley & Sons, 2007. (ISBN 978-0471931515).

[47] G. Henzold, Geometrical Dimensioning and Tolerancing for Design, Manufacturing and Inspection: A Handbook for Geometrical Product Specification Using ISO and ASME Standards, Butterworth-Heinemann, 2006 (ISBN 978-0750667388).

[48] K. Yang, S. Basem, B. El-Haik, Design for Six Sigma: A Roadmap for Product Development, McGraw-Hill New York, 2003 (ISBN 978-0071412087).

[49] R.M. Lawson, J. Richards, Modular design for high-rise buildings, Proceedings of the Institution of Civil Engineers-Structures and Buildings 163 (2010) 151–164, https://doi.org/10.1680/stbu.2010.163.3.151.

[50] P. O'Connor, A. Kleyner, Practical Reliability Engineering, John Wiley & Sons, 2012 (ISBN 978-0470979815).

[51] R. Cvetko, K.W. Chase, S.P. Magleby, New metrics for evaluating Monte Carlo tolerance analysis of assemblies, http://adcats.et.byu.edu/WWW/Publication/98-2/CvP1-2col_6=30=98.PDF, (1998) (Feb 2, 2019).

[52] F.J. Fuentes, V. Trouvé, J. Cordier, J. Reich, Methodology for dimensional variation analysis of ITER integrated systems, Fusion Engineering and Design (2016), https://doi.org/10.1016/j.fusengdes.2016.03.006.

[53] AISC, Specification for structural steel buildings (ANSI/AISC 360-10), https://www.aisc.org/globalassets/aisc/publications/standards/a360-16-spec-and-commentary.pdf, (2010) (Feb 2, 2019).

[54] M. Shahtaheri, C.T. Haas, T. Salimi, A multi-dimensional joint confidence limit approach to mixed mode planning for round-the-clock projects, Eng. Constr. Archit. Manag. 24 (2017) 40–60, https://doi.org/10.1108/ECAM-10-2015-0165.

[55] M. Lawson, R. Ogden, C. Goodier, Chapter 16: construction issues in modular systems, Anonymous Design in Modular Construction, CRC Press, 2014, pp. 211–224 (ISBN 978-0415554503).

[56] P. Sharafi, M. Mortazavi, B. Samali, H. Ronagh, Interlocking system for enhancing the integrity of multi-storey modular buildings, Autom. Constr. 85 (2018) 263–272, https://doi.org/10.1016/j.autcon.2017.10.023.

[57] J. Kim, K. Kim, J. Lee, J. Jeong, Generation of assembly models from kinematic constraints, Int. J. Adv. Manuf. Technol. 26 (2005) 131–137, https://doi.org/10.1007/s00170-004-2231-3.

[58] P.J. Drake, Dimensioning and Tolerancing Handbook, McGraw-Hill, New York, 1999 (ISBN 978-0070181311).