2019
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of each answer sheet represent the problem number that you answer (P 1, P 2, P 3, P 4) and also the class of the course (master M, doctor D) that you are applying. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet.

6. You may use the blank sheets of the problem booklet for rough papers without detaching them.

7. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

I. Consider a pulley of radius a and mass M having a uniform volume density, which can rotate freely without friction around the fixed point O, as shown in Fig. 1.1. Weight 1 of mass m_1 and weight 2 of mass m_2 are suspended from the pulley by a string, and then gently released. Assume that the pulley and weights are stationary at the initial state. g is the acceleration due to gravity, and we assume that $m_2 > m_1$. Mass, thickness, and elongation of the string, as well as the dimension of weights are negligible. There is no slipping between the pulley and the string. Answer the following questions.

1. Derive the moment of inertia of the pulley around the fixed point O. Also write the derivation process.
2. When the weight is released at time $t=0$, derive the velocity of weight 1 along the y direction, as a function of time t.

II. A cylinder of mass M, outer diameter $2a$, and inner diameter a rotates with angular velocity ω_0 around the central axis, as shown in Fig. 1.2. Consider the motion when the cylinder is gently put on a horizontal plane with rough surface. Answer the following questions. Here, the cylinder is a rigid body having a uniform volume density, the coefficient of kinetic friction between the cylinder and the horizontal plane is μ, and the acceleration due to gravity is g. Also, the energy loss of the cylinder when rolling on the plane without slipping is negligible.

1. Derive the moment of inertia of the cylinder around the central axis. Also write the derivation process.
2. When the cylinder is put on the plane at time $t=0$, it starts rolling with slipping. Obtain the velocity of the cylinder's center of gravity and the angular velocity around the cylinder's center of gravity, as functions of time t, when the cylinder is rolling with slipping.
3. Derive the time t_1 and distance x_1 which are required for the cylinder to start rolling without slipping.
4. A rough slope forming an angle θ with the horizontal plane is located far from the distance x_1. Calculate the maximum height to which the cylinder can rise by rolling without slipping. Assume that

θ is small enough that the cylinder can move from the horizontal plane to the slope while maintaining contact.

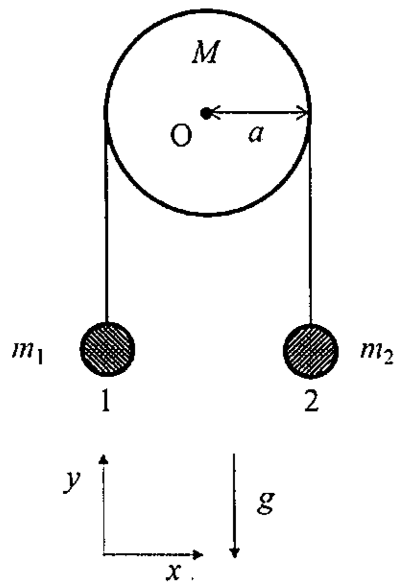

Figure 1.1

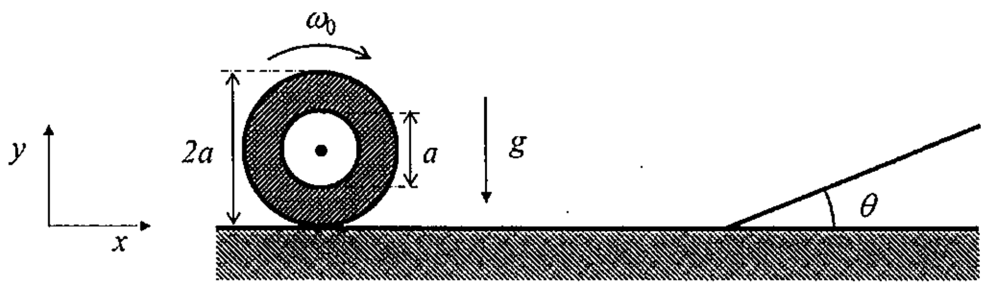

Figure 1.2

Problem 2

Consider electric and magnetic fields induced by electric charges, magnetization, and currents in vacuum. Let the dielectric constant and magnetic permeability of vacuum be ε_0 and μ_0, respectively. You can neglect gravity. Answer the following questions.

I. As shown in Fig. 2.1, electric charges are distributed over a disk with a hole (inner radius: a, outer radius: b) with a uniform surface density $-\sigma\,(\sigma > 0)$ in the xy plane. Let the center of the disk be O.

1. Find the electric potential and field at the point $\mathrm{P}\,(0,0,z)$.
2. Express the electric field obtained in I.1 in the case where $z >> b$ and briefly explain the physical meaning of this expression.
3. Show that the electric field obtained in I.1 is equal to that induced by the infinite plane sheet of uniform electric charge when $a \to 0$ and $b \to \infty$.
4. A particle with negative electric charge $-q\,(q > 0)$ and mass m is put at the point $\mathrm{C}\,(0,0,c)\,(c > 0)$ which is very far from O and given an initial velocity of $\overrightarrow{v_0} = (0,0,-v_0)\,(v_0 > 0)$. Find the z-coordinate of the particle when it is closest to O. Here, v_0 is small enough that the z-coordinate of the particle z_c is always much larger than b $(z_c >> b)$ during the motion of the particle. After a sufficient period of time has elapsed (i.e. time $t \to \infty$), the velocity of the particle converges towards a certain $\overrightarrow{v_c}$. Find $\overrightarrow{v_c}$.
5. A particle with positive electric charge $q\,(q > 0)$ and mass m is put at the point $\mathrm{D}\,(0,0,d)\,(d \approx 0)$ near O and then gently released. Explain the motion of this particle and find its period. You can neglect electromagnetic wave emission from the particle during the motion.

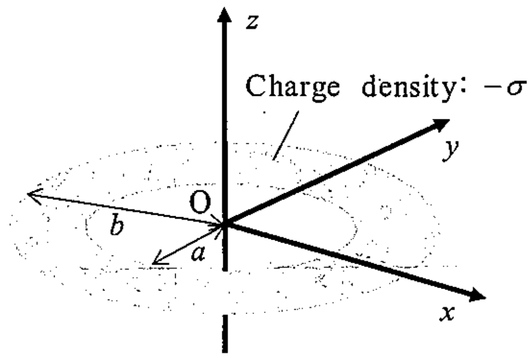

Figure 2.1

II. Consider a magnetic cylinder with an inner radius a, outer radius b, thickness t and rectangular cross section in vacuum.

1. As shown in Fig. 2.2, a magnetic cylinder is magnetized along the central axis of the cylinder with constant magnitude M. Explain in what part of the cylinder the magnetization currents (the currents inducing magnetization) flow and in what directions.
2. As shown in Fig. 2.3, a magnetic cylinder is magnetized along the circumferential direction of the cylinder with constant magnitude M. Find the magnetic field and magnetic flux density in and outside the cylinder.
3. As shown in Fig. 2.4, a circular solenoid is formed by winding a conductive wire uniformly around this magnetic cylinder (total number of coils: N). Find the total magnetic flux penetrating the cross section of the magnetic cylinder when the current I flows in the solenoid. We assume that the cylinder is not magnetized before the current flows and neglect leakage of the magnetic flux. Let the magnetic permeability of the cylinder be μ.
4. Assume the magnetic flux density and magnetization are uniform in the circular solenoid. Find the magnetic flux density and magnetization in the cylinder using the average circumference of $2\pi(a+b)/2$. In addition, show that this magnetic flux density is equal to that calculated from the magnetic flux in II.3, when the inner radius is much larger than the difference between inner and outer radii (i.e. $a \gg b-a$) by considering the assumption that $a \gg b-a$ and the

magnetic flux density is uniform in the cylinder. If necessary, you may use the approximation $\ln(1+x) \approx x$ when $|x| \ll 1$.

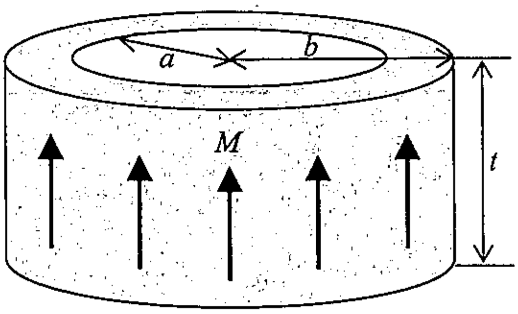

Figure 2.2

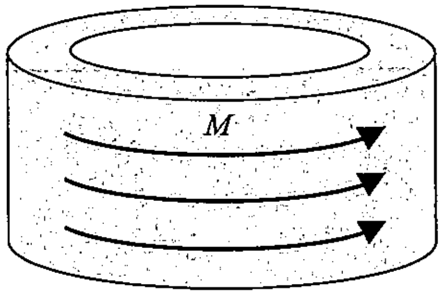

Figure 2.3

Total number of coils: N

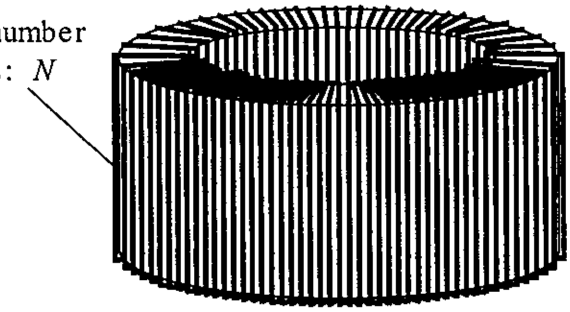

Figure 2.4

Problem 3

I. The following equation shows the first law of thermodynamics.

$$d'Q + d'W = dU. \tag{1}$$

Here Q is the amount of heat supplied to the system, W is the amount of work done on the system, and U is the internal energy. The specific heat at constant volume, C_V, is expressed as,

$$C_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V. \tag{2}$$

Here, n, V, and T are the number of moles, the volume, and the temperature, respectively. Let P and C_P be the pressure and the specific heat at constant pressure, respectively. Answer the following questions.

1. Show that the following relation holds true for an ideal gas in a quasi-static adiabatic process.

$$nC_V dT + PdV = 0. \tag{3}$$

2. Show that the following relation holds true for an ideal gas in a quasi-static adiabatic process using equation (3).

$$TV^{\gamma-1} = \text{constant}, \tag{4}$$

where γ is the ratio of specific heat ($\gamma = C_P / C_V$). You may use the fact that the relationship between C_P and C_V can be expressed as $C_P - C_V = R$ (R: the gas constant) for an ideal gas.

II. An ideal gas of n moles is contained in a cylinder with a constant cross-section Z and the gas is manipulated according to the quasi-static thermal cycle, A→B→C→D→A, depicted in Fig. 3.1. Here, P, V, and T are the pressure, the volume, and the temperature, respectively. The processes A→B and C→D are isothermal and the processes B→C and D→A are adiabatic.

(Volume, Temperature) of the gas at state A, B, C, and D, are (V_1, T_1), (V_2, T_1), (V_3, T_2), and (V_4, T_2), respectively. Assume that the friction between the cylinder and the piston is negligible during the adiabatic processes and isothermal expansion process, whereas a constant friction force f is applied between the cylinder and the piston as shown in Fig. 3.2 during the isothermal compression process (C→D). All the work done by the friction force is absorbed by the contained ideal gas. Here R is the gas constant. Answer the following questions.

1. Find the amount of heat absorbed by the gas, Q_1 (>0), during the process A→B.
2. Find the amount of heat absorbed by the gas, Q_2 (<0), during the process C→D.
3. Find the efficiency η ($\eta = (Q_1 + Q_2)/Q_1$) of the cycle A→B→C→D→A, using n, R, f, Z, T_1, T_2, V_3, and V_4.

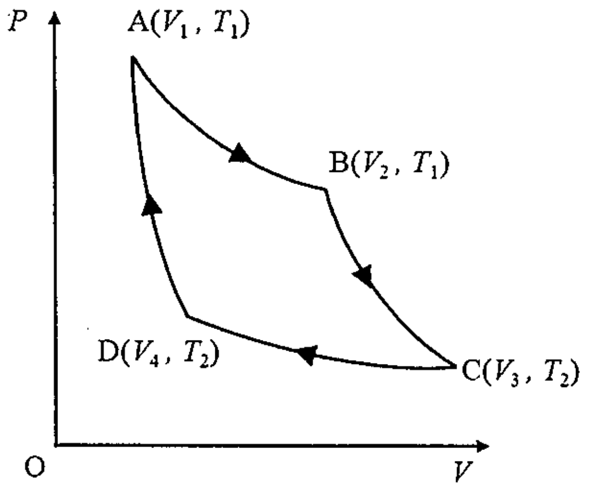

Figure 3.1

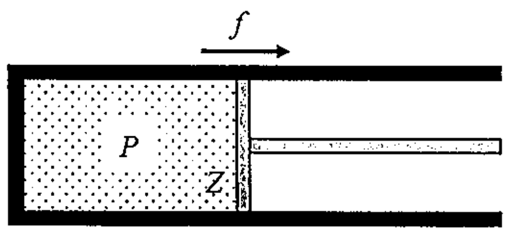

Figure 3.2

III. Consider a reversible cell, where the volume change is negligible during the reaction and the work is done only by the charges. The amount of work done by the cell is expressed as Edq, with electromotive force E and charge q. In the reversible cell, the first law of thermodynamics is expressed as,

$$dU = TdS - Edq, \tag{5}$$

where U is the internal energy, T is the temperature, and S is the entropy. The Helmholtz free energy is defined by

$$F = U - TS. \tag{6}$$

Answer the following questions.

1. Show that the following relations between Helmholtz free energy F, electromotive force E, and entropy S hold true. You may consider that F can be expressed with independent variables q and T.

$$E = -\left(\frac{\partial F}{\partial q}\right)_T, \tag{7}$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_q. \tag{8}$$

2. Show that the following relation about the internal energy change holds true.

$$\left(\frac{\partial U}{\partial q}\right)_T = -E + T\left(\frac{\partial E}{\partial T}\right)_q. \tag{9}$$

3. Assume the electromotive force of the reversible cell is expressed as,

$$E = 0.49 + 0.0002T \text{ [V]}. \tag{10}$$

Calculate the amount of heat absorbed by the cell when the cell is discharged at 200 mA for 10 seconds at 300 K. Note that charge movement dq during the discharge is expressed as $dq = It$ with current I and time t. Here the temperature variation and the electromotive force variation of the cell are negligibly small during the discharge.

Problem 4

Consider the principle of the acceleration sensor. Here, a hollow rigid box containing a body (regarded as a point mass) of mass m is connected to the box by a spring (an element generating a restoring force proportional to the displacement from the natural length: linear coefficient k) and a damper (an element generating a resisting force proportional to the velocity of the body: linear coefficient c) as shown in Fig. 4.1. The box and the body do not rotate, and move only vertically. The coordinate of the box in space and the coordinate of the body relative to the box from the natural length of the spring are denoted x and y, respectively. Note that the time is t and no gravity is considered.

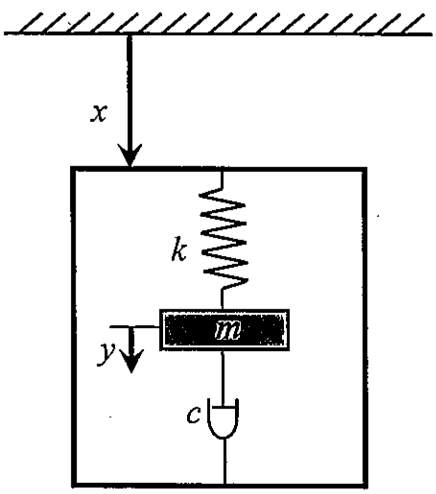

Figure 4.1

I. Express the inertial force acting on the body using m, x, y, and t.

II. The restoring force on the body by the spring can be expressed as $-ky$ and the resisting force acting on the body by the damper can be expressed as $-c\frac{dy}{dt}$. Derive the equation of motion of the body.

III. Consider an oscillation of the box with amplitude a and angular frequency ω, expressed as $x = a\cos\omega t$. When only the steady-state response is considered, answer the following questions.

1. Express the response y of the body in the form $y = y_0 \cos(\omega t - \beta)$. Here, express y_0 and β using γ and μ as defined below, and a. Note that $\omega_0 = \sqrt{\frac{k}{m}}$.

$$\gamma = \frac{c}{2\sqrt{mk}}, \tag{1}$$

$$\mu = \frac{\omega}{\omega_0}. \tag{2}$$

2. Show the relationship between y_0 and μ by a diagram, in the case when $c = 0$ and in the case when $c = 2\sqrt{mk}$.

3. When ω is large enough compared to ω_0, approximate y_0 and β. Based on this result, deduce a method to estimate the response x of the box using the response y of the body.

4. When ω is small enough compared to ω_0, approximate y_0 and β. Based on this result, deduce a method to estimate the acceleration $\frac{d^2x}{dt^2}$ of the box using the response y of the body.